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Three essays in financial frictions

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Three essays in financial frictions

by

Sicheng He

A dissertation submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Economics

Program of Study Committee:
Juan Carlos Cordoba, Major Professor
Joydeep Bhattacharya
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Quinn Weninger

The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this dissertation. The Graduate College will ensure this dissertation is globally accessible and will not permit alterations after a degree is conferred.

Iowa State University

Ames, Iowa

2019

ii

DEDICATION

To my family

TABLE OF CONTENTS

ACKNOWLEDGMENTS

I would like to thank my advisor Juan Carlos Cordoba. His guidance and advice have been invaluable. It is impossible for me to really understand how to be a real researcher without his help.

I would also like to thank Joydeep Bhattacharya, Ivan Rudik, Rajesh Singh, Quinn Weninger and Gary Lyn for their help. They have given me a lot of good advice about my research.

ABSTRACT

My dissertation studies how financial frictions affect economy, especially macroeconomy.

Chapter 2 studies the potential for rational bubbles in the innovation sector to affect long term economic growth. We show that stock market prices of R&D firms could include a bubble component when credit constraints are present. Bubbles are self-sustained in equilibrium by a "liquidity" premium that originates when credit constraints are relaxed. Bubbles expand borrowing and production capacity of R&D firms, stimulate innovation and increase the growth rate. Bubbles are magnified by tighter credit constraints and scarce investment opportunities. Finally, we show that bubbles can create permanent reallocation effects benefiting the innovation sector over other sectors.

Chapter 3 uses a generalized Kiyotaki and Moore model (1997) with collateral and cash-in-advance constraints to study the effects of financial and non-financial crisis and the effects of monetary policy both in the short and the long run. We then characterize optimal monetary policy in the Ramsey sense. We find that in the long run, the optimal monetary policy drives the social, but not the individual, shadow price of the collateral constraint to zero. This translates into a generalized version of the Friedman's rule, one that takes into account the degree of credit tightening. In the short run, optimal monetary policy is counter-cyclical, significantly offsetting the effects of financial shocks and reducing the welfare loss of the shocks.

Chapter 4 studies the dynamics of blockchain innovation, adoption and competition in the global payment industry in the presence of a traditional technology. We build a theoretical model with network effects to study the possible evolution path of the payments industry, how a particular technology can gain and lose its market share and whether there exist some

technology which can maintain its dominant power. We also study the role of bubbles, and show that they have positive and negative effect on the social welfare.

CHAPTER 1. GENERAL INTRODUCTION

When economists discuss how certain phenomenon affects economy, they often ask questions such as "How does this phenomenon affect the economic growth?""How does this phenomenon generate business cycles?""What can government do to reduce the negative effect?""What effects do this phenomenon have on certain industry?" In my dissertation, I try to answer these questions with financial frictions.

Chapter 2 is Endogenous Growth, R&D, Credit Constraints and Bubbles. This chapter focus on how credit constraints affect innovation and economic growth, why bubbles exist in innovation sector and how bubbles related with credit constraints. In this paper, I introduce credit constraints into R&D sector which plays important role in many endogenous growth model. I find there are multiequilibria in equity value when credit constraints are binding and bubbles are possible to exist in such economy. To the extent of my knowledge, this is the first paper which find bubbles exist in standard endogenous growth model with infinite periods living households. I find that bubbles exist in this model because it can reduce liquidity mismatch in R&D sector caused by credit constraints. Thus, bubbles help R&D sector to get fund. This effect alone with reallocation effect encourage innovation and economic growth.

I also study Optimal Monetary Policy with Collateral Constraints with Juan Carlos Cordoba and this is Chapter 3 of my dissertation. In this study, we study how an economy with collateral constraints and cash in advance constraints can be affected by financial shocks, technology shocks and money shocks. We also use Ramsey policy to study both the long term and short term optimal monetary policy. We find that in the long run, the optimal monetary policy drives the social, but not the individual, shadow price of the collateral constraint to zero. This translates into a generalized version of the Friedman's rule, one that takes into account the degree of credit tightening. In the short run, optimal monetary policy is counter-cyclical, significantly offsetting the effects of financial shocks and reducing the welfare loss of the shocks.

Chapter 4, Blockchain Innovation in Global Payments: Network Effects, Creative Destruction and Bubbles, focuses on how financial frictions can be reduced in a certain

industry with certain technology. For a long time, global payments are costly and inefficiency. In recent years, people are trying to solve this problem with new blockchain technology. This paper trying to study how the evolution of this innovation may be and how it affect the industry. However, both global payments and blockchain technology face network effects. Although there are many papers about network effects, there are very few papers about introducing new technologies with network effects. This paper builds a theoretical model with network effects to study the possible evolution path of the payments industry, how a particular technology can gain and lose its market share and whether there exist some technology which can maintain its dominant power.

CHAPTER 2. ENDOGENOUS GROWTH: INNOVATION, CREDIT CONSTRAINTS, AND STOCK PRICE BUBBLES

2.1 Introduction

Innovation drives modern economic growth (Romer 1990, Grossman and Helpman 1991, Aghion and Howitt 1992). At the same time, innovation and technological progress often correlate with bubbles. For example, in his classic book, Shiller (2015) finds that there was rapid economic growth and widespread dissemination of technological innovations in the 1920s which led to bubbles burst later in the Great Depression. Similarly, Sorescu et al. (2018) study 51 major innovations introduced between 1825 and 2000, from steam engine train to smartphone. They detect bubbles in approximately 73% of the innovation. A well-known instance where innovation and bubbles coincided was the so called dot-com boom. In the late 1990s, between 7,000 to 10,000 new Internet companies were founded seeking to take advantage of the new possibilities open by the internet. This was a period of rapid innovation and expanded variety of internet products (Wang 2007). At the same time, the Nasdaq Composite stock market index rose 400%.

A second feature of innovation, or research and development (R&D), activities is that they often face credit constraints (Brown, Martinsson and Petersen 2012). Studies have found that this is particularly important for small and medium size firms (Beck and Demirguc-Kunt 2006). Credit constraints are likely due to asymmetric information and lack of collateral. Information asymetries in turn arise from the underlying characteristics of innovation since insiders have better information about the real chances of succees.

This paper develops a theory of economic growth driven by innovation, innovators facing collateral constraints, households acting as venture capitalists, and stock prices of R&D firms determining the extent of R&D activities. As we show, our model can deliver rational bubbles that are sustained in equilibrium by a "premium" that arises when collateral constraints are relaxed. Our model is able to explain why bubbles can exist as an essential part of a growing economy. We use the model to show the potential effects of bubbles on innovation and long term growth.

Our baseline follows endogenous growth models with expanding varieties first developed by Romer (1990). Romer's model is widely used to study issues of innovation and endogenous growth. It has a final goods sector with a representative firm, a monopolistic competitive intermediate goods sector and a R&D sector. The final goods sector and intermediate goods sector in this paper are standard. Competitive final goods producers use intermediate goods, each producer of an intermediate good is a monopolist who produces a differentiated variety which rights of production are purchased from the R&D sector.

Our major difference with standard variety models of endogenous growth is in the R&D sector which is subject to credit constraints as in Kiyotaki and Moore's (2005) and Miao and Wang (2018). We assume there are a continuum of R&D firms, owned by households, which use both capital and labor to create new varieties that are sold to intermediate producers. R&D firms have random investment opportunities which allow them to transform output into capital which is useful for R&D production. Firms use their revenue from selling their patents and intratemporal debt from firms without investment opportunities to fund their investment. R&D firms face credit constraints which are related to their equity value (feature 2). In the event of default, lenders take over the firm but some capital is loss in the process. Thus, debts cannot be larger than the taking over value.

We show that the model exhibit multiple equilibria. In particular, there are two possible balanced growth paths (BGP), a bubbleless one and a bubbly one. Bubbles exist in R&D firms' stock price for one of the two BGP (feature 1). In particular, there may be two components existing in stock prices. One is related to the future revenue of innovation and investment activities which determines the stock fundamental value. The other one is not related to future income directly. Even a firm without any capital can still have high stock price. This part is defined as bubble.

Our baseline model not only reflects two features in innovation in an endogenous growth model, but also provides a mechanism about how bubbles affect credit constraints and innovation. The most important result of the model is bubbles stimulate innovation

when there are credit constraints. There are two effects when there are bubbles. The direct one is positive while the indirect one is negative. The direct effect is the existence of bubbles increases the value of collateral directly so firms can borrow more from lenders when they have investment opportunities thus the R&D industry has more investment and produce more blueprints. This is similar with crowd in effects which have been studied in other literature (Hirano and Yanagawa 2017, Miao and Wang 2018).

At the same time, since stock value is also related with capital which we define as fundamental value of stock, looseness of credit constraints reduce the demand of capital. Thus capital price decreases and the value of collateral decreases. What is more, the decrease of capital price also decreases the capital revenue from producing and selling blueprints so it is a negative effect on investment. These two negative effects offset some of the crowd in effect. However, the direct effect is the dominant one thus bubbly BGP has higher growth rate than bubbleless BGP. It is worth to mention that bubbles' size are related with credit constraints. The tighter credit constraints are binding, the bigger the bubbles are. The effects of bubbles will also be bigger when credit constraints are tighter.

Since there are effects in different directions, bubbles can affect the inside value of the firm differently from the outside value. We find that bubbles typically increase the outside value of the firm, the value to the lenders in the event of default. Thus, lenders would like to lend more to the borrowers. But bubbles may increase or decrease the inside value of the firm, the equity value to the owners. Figure 1 explains the results. In our model, there is cost for lenders taking over the firm when borrowers default so lenders only get the firm with ξ of capital. k_1 and k_2 are borrowers' capital in two cases. V^{bb} and V^{nb} are bubbly and bubbless value of the firm given capital k. In both cases, the stock price the lenders can get is higher with bubbles. However, it is undetermined whether bubbles increase the value (k_2 case) or decrease the value (k_1 case). This is different with Miao and Wang (2018). In their paper, bubbles invariably increase the stock price while in our model it mainly increase the value, to the lenders, in the event of default. The reason why rational bubbles exist and are sustained in one of the equilibria is due to their role
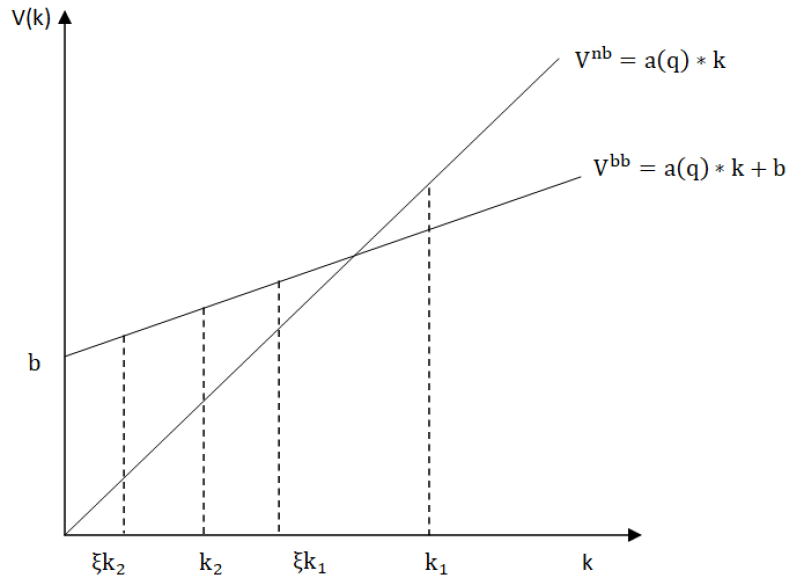
Figure 1: The relationship between equity value and capital

relaxing borrowing constraints which provides the underlying liquidity premium. Even though the revenue from growth of the bubbles is less than interest rate, people still accept the bubbles because they get additional investment revenue since bubbles increase collateral.

We also study what happens when bubbles burst. We find that investment is decreasing after the burst of bubbles due to the credit constraints so capital gradually converges to the bubbleless BGP. However, capital price acts much more rapidly. The mechanism is just the same as what we have described before.

We then extend our model to study the reallocation effects of bubbles. We show that besides the effects in the baseline model, stock price bubbles in R&D sectors also attract more labor into R&D sectors which further help the innovation and economic growth.

Our model suggests bubbles in innovation sector is good for the growth so government shall not make them burst without careful conditions. The existence of bubbles means innovation sector may face tight financial constraints. Thus, the right thing government shall do is to reduce the financial frictions and help innovation firm to get enough funds when there are bubbles in innovation sector.

In the remaining part of this section, we review related literature and discuss our contribution. In section 2, we introduce the baseline model. Section 3 is about analysis of equilibria. In section 4, we derive BGPs for both bubbly economy and bubbleless economy and compare the difference. We then give mechanism how bubbles work. In section 5, we study the dynamics of bubbles. We study the dynamics around the BGP and what happens when bubbles burst. Extensions are in section 6 where we have stochastic bubbles and reallocation effects. Section 7 is conclusion.

2.1.1 Related Literature

This paper is mostly related with literature seeking to understand the connection between growth and bubbles. Tirole (1982) finds that bubbles do not exist in standard infinite period complete market models because the existence of bubbles lead to the violation of transeversality conditions. However, economists find that bubbles exist in some incomplete market models. Overlapping Generations (OLG) models attract a lot of attention among such models. Samuelson (1958) is an early study using OLG model with money. In his study, money is pure bubble helping to solve lacking of debt market. Tirole's work (1985) is a fundamental study for bubbles in OLG models and has inspired a large literature. Among them are some papers using OLG models to study growth with bubbles (e.g., Caballero, Farhi and Hammour 2006, Martin and Ventura, 2012). They find bubbles can crowd savings away from investment but they also find bubbles may also provide additional asset and encourage investment. However, These OLG papers have some disadvantages. The market incompleteness relies on the lack of market between generations which are not the reasons lead to the existence of bubbles. They are also not suitable to do realistic quantitative explorations as Hirano and Yanagawa (2017) point out.

Besides these standard OLG models. Olivier (2000) uses a continuous-time OLG model. Olivier's study is the one which is close to our paper. He finds that bubbles in R&D sector can benefit the growth while bubbles in other type of assets may harm the economic growth. However, Olivier's R&D sector is very simple and does not reflect the

features of innovation. Also, bubbles are not generated by any properties of R&D sector. Bubbles in R&D sectors and other sectors are generated by demographic reasons and do not have any difference.

In recent years, there are some papers using infinitely lived agents model to study bubbles. Hirano and Yanagawa (2017) build a model with financial frictions and heterogeneous investments. However, there is no innovation sector so the technology is exogenous and we cannot know the relationship between bubbles and innovation. Secondly, they use a useless asset as bubbles. Although this kind of fiat bubbles have long tradition in literature, it does not reflect what happens in innovation. For example, dot-com bubbles happen in stock price and has no relationship to any useless asset. This useless asset also leads to strong crowd out effects as in Olivier (2000). Thirdly, their model can only be used to study bubbles in countries with intermediate level of financial frictions. Bubbles do not exist in financially underdeveloped or well-developed countries. This is clearly not true in innovation because we have already seen some bubbles in the R&D sector of United States which is one of the most financially developed countries in the world. Miao and Wang (2014) build a model using stock value as collateral and have two sectors. One sector has externality while the other one does not. In their model, bubbles can relax collateral constraints. However, it does not have innovation sector so the technology of the model is exogenous.

Our paper, however, solve all the problems we discuss before. Our model follows the work by Miao and Wang (2018). Although it is not an endogenous growth model, Miao and Wang provide a novel way to think about bubbles. They use an infinitely lived agents model with credit constraints and heterogeneous investment opportunities to show that bubbles can exist in standard infinitely lived agents model with incomplete market and not violate transeversality conditions by reducing liquidity mismatch when there are investment opportunities. Our paper has the similar setting in R&D sector and find that bubbles not only affect steady state, but also increases BGP of an endogenous growth model by stimulating innovation. To the extent of our knowledge, there is no paper studying bubbles in an endogenous model with a well defined R&D sector before.

Since bubbles in our paper exist in stock price rather than on an useless asset, our paper follows Oliver (2000) and bubbles will always help innovations rather than hurt the economic growth. But our paper shows that even bubbles may not increase stock price directly, it still stimulate innovations. The mechanism in our paper are related more with innovation rather than demographics which is a exogenous variable in innovation. Also, our paper find that although Miao and Wang's mechanism does exist, bubbles have more complex effects than Miao and Wang's mechanism. Besides their positive effects, there are also some negative effects to offset it. Failing to consider negative effects results in overestimate the benefit of bubbles. Thus, bubbles increase stock value which can be acquired by lenders rather than borrowers. In our extension section, we also find social resource has been reallocated to R&D sector when there are bubbles. This is a new effect which help the growth of the economy. Our model is rather robust with financial development. For underdeveloped countries, bubbles are always able to exist. Even for the most developed countries, bubbles are still able to exist if the investment opportunities do not come too often. Thus, our model can be used in study bubbles in different countries.

Besides the papers in growth and bubbles. Our paper is related with papers in different fields. First of all, this paper is related with papers studying endogenous growth with innovation. Relationship between economic growth and innovation have been studied by economists for a long time both in empirical way and theoretical way. Economists find economic data provides evidence that innovation and growth are positively related (Griliches and Lichtenberg 1984, Zachariadis 2003). There are also a large number of papers focus on studying growth and innovation in theoretical way. Romer (1990) builds a model with a R&D sector where technological innovation is in the form of expanding varieties created by labor in R&D sectors and existing knowledge. It is the R&D keeps the economy growth in long term. Grossman and Helpman (1991) and Aghion and Howitt (1992) both build model to study how R&D which improve products quality have effects on growth. Both empirical and theoretical studies find that R&D has strong effects on

economic growth. Our paper extend studies in this field by introduce credit constraints and bubbles into R&D sector.

Our paper is also related to papers studying credit constraints. The seminal work of Kiyotaki and Moore (1997) introduces collateral constraints into general equilibrium and finds collateral constraints have significant effect on the whole economy. Numerous studies follow Kiyotaki and Moore to study the effect of collateral constraints (e.g., Cordoba and Ripoll 2004, Iacoviello 2005 and Liu, Wang and Zha 2013). However, most of these studies focus on business cycles. There are few theoretical papers study innovation with credit constraints. Amable, Chatelain and Ralf (2010) is one trying to study credit constraints with R&D. They find that patents created from R&D process can be used as collateral to reduce the negative effect of collateral constraints. Our study provides a noval way to think of credit constraints and R&D.

2.2 The Baseline Model

Since our model is rather complicated, we use figure 2 to help us introduce our model before we describe it in detail. Arrows in figure 2 indicate flow of resource and goods. The representative household hold shares of firms in R&D sectors and provides labor to R&D firms. The household also get income by receiving dividends and wages. R&D firms use capital and labor to produce new patents and sell patents to intermediate goods. After that, a firm in R&D sector has investment opportunity with probability π. Those who have investment opportunities borrow from those without investment opportunity and invest but they are constrained by credit constraints. Final goods are transformed into new capital when firms invest and firms trade capital after investment stage. After buying patents from R&D firms, intermediate goods producers produce intermediate goods by using final goods. They sell intermediate goods to final goods producer who use intermediate goods to produce final goods. Besides the flow of resource, figure 2 also point out R&D firm j cannot borrow more than firm's discounted value. When there are bubbles, firm j's discounted value is greater than without bubble.
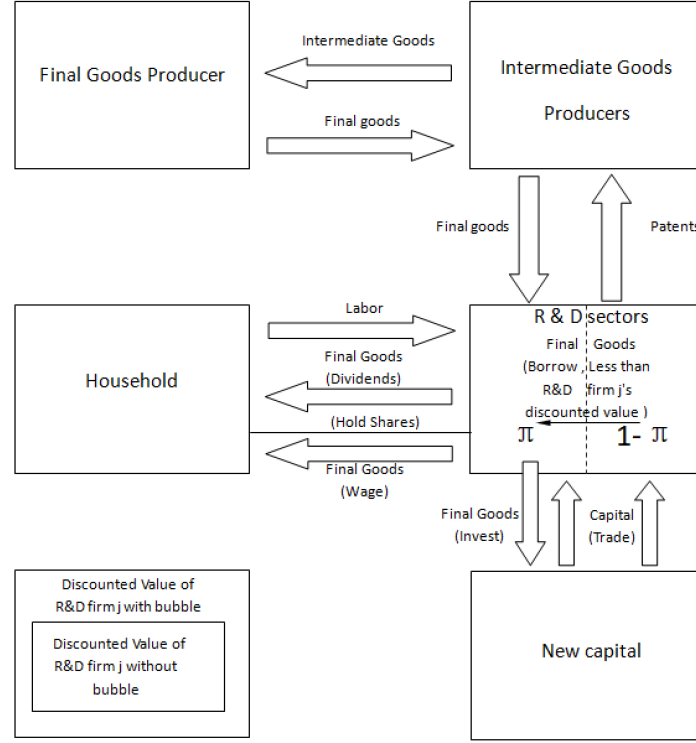
Figure 2: Bird's eye view of the baseline model

2.2.1 Households

There is a representative household in our model who has a standard utility function

$$\sum_{t=0}^{\infty} \beta^t \ln C_t$$

where β is the discount rate and C_t is the consumption in period t. Household provides all its labor inelastically every period and aggregate labor supply is normalized to 1. Household trades stocks of firms in R&D sectors every period and also receive dividends from stocks it holds. Household uses wages and income from trading stocks to buy consumption and do not have any other way to save. Thus, Household faces budget constraints

$$C_t + \int \left(V_t^j - D_t^j \right) \psi_{t+1}^j dj = \int V_t^j \psi_t^j dj + W_t$$

where W_t is the wage rate, V_t^j, D_t^j are R&D firm j's cum-dividend equity value and dividend and ψ_t^j is household's holdings of firm j's shares.

Transversality conditions are

$$\lim_{T \to \infty} \beta^T \frac{V_T^j \psi_T^j}{C_T} = 0$$

Thus, the representative household maximizes its utility function while budget constraints and transeversality conditions are satisfied.

We define the growth rate of consumption g_{t+1}^c

$$g_{t+1}^c = \frac{C_{t+1}}{C_t} - 1$$

and

$$\rho_{t+1} = \beta \frac{C_t}{C_{t+1}} = \beta \left(1 + g_{t+1}^c\right)^{-1}$$

where ρ_{t+1} is the stochastic discount factor in asset pricing literature.

2.2.2 Final Goods Producer

In our model, there is only one kind of final goods. Let the final goods are numeraire and all consumptions, investment and inputs are using final goods. For simplicity we assume there is only one representative firm produce final goods and it is a price taker. The final goods producer uses intermediate goods to produce and the technology is

$$Y_t = A \int_{n=1}^{N_t} (X_t^n)^\sigma \, dn, 0 < \sigma < 1$$

Here N_t is total number of varieties in period t and X_t^n is the amount of intermediate goods n the final goods producer uses. A denotes the technology of final goods producer.

We use P_n^t to denote the price of intermediate goods n. Profit maximization problem of the final goods producer is

$$\max Y_t - \int_{n=1}^{N_t} P_t^n X_t^n dn$$

subject to the production function. It is easy to solve profit maximization problem and we have the demand function for intermediate goods n

$$P_t^n = \sigma A \left(X_t^n\right)^{\sigma-1}$$

2.2.3 Intermediate Goods Producers

Intermediate good X_t^n is produced in competitive monopolistic markets. To produce an intermediate goods n, an intermediate goods producer has to pay a patent fee η_n to the R&D firm who creates blueprint n first. After paying the patent fee, the intermediate goods producer can produce any amount of intermediate goods at any periods. The technology of intermediate goods producer is it can transform one unit of final product to one unit of X_t^n. Thus his profit is

$$\left(P_t^n - 1\right) X_t^n$$

Since we have already had intermediate goods n's demand function, we can find the price intermediate goods producer of goods n set

$$P_t^n = \frac{1}{\sigma}$$

and the amount the producer produces

$$X_t^n = \sigma^{\frac{2}{1-\sigma}} A^{\frac{1}{1-\sigma}} \tag{1}$$

Then the profit of producing goods n every period is

$$\left(\frac{1-\sigma}{\sigma}\right) \sigma^{\frac{2}{1-\sigma}} A^{\frac{1}{1-\sigma}}$$

. Since we know that it is competitive monopolistic markets, the discounted total profits from selling goods n must be equal to the cost of buying patent to produce goods n, which means $\sum_{s=t}^{\infty} \rho\left(s,t\right) \left(\frac{1-\sigma}{\sigma}\right) \sigma^{\frac{2}{1-\sigma}} A^{\frac{1}{1-\sigma}} = \eta_n$. Here $\rho\left(s,t\right) = \prod_{v=t+1}^{s} \left(\rho_{v+1}\right)$ if $s \neq t$,

$\rho(s, t) = 1$ if $s = t$. Since only variables in η_n are time variables, patents created in the same period have the same price. This result gives us

$$\sum_{s=t}^{\infty} \rho(s, t) \left(\frac{1-\sigma}{\sigma} \right) \sigma^{\frac{2}{1-\sigma}} A^{\frac{1}{1-\sigma}} = \eta_n = \eta_t \tag{2}$$

2.2.4 R&D Sector

There are a continuum of firms $j \in [0, 1]$ in R&D sector. In every period, there are three stages. We first briefly introduce the three stages and then provide details. At the first stage, firms hire labor to create new blueprints and sell them to intermediate producers as patents. During the second stage, some firms have opportunities to invest and get new capital. They can use their own fund or loans from other firms to invest. At the third stage, firms trade capital with each other.

At the beginning of period t, firm j in R&D sector has K_t^j amount of capital it accumulated at the end of period $t - 1$. Thus capital at the first stage is given. It then hires L_t^j amount of labor. Technology for firm j uses both capital and labor to create new blueprints. T_t^j is the amount of new blueprints created by firm j in period t. Current technology level (current amount of blueprints N_t) also has effect on the innovation process. The production function of R&D firm j is

$$T_t^j = Z \left(K_t^j \right)^{\alpha} \left(N_t L_t^j \right)^{1-\alpha}$$

where Z is an exogenous parameter. This technology of innovation means that technology has spillover effects. Every invention benefits future invention by increasing labor productivity. This property is common setting in endogenous growth models. Capital depreciates at rate δ every period. Capital return of producing new blueprints at period t is

$$r_t^j K_t^j = \max_{L_t^j} \eta_t Z \left(K_t^j \right)^{\alpha} \left(N_t L_t^j \right)^{1-\alpha} - W_t L_t^j \tag{3}$$

It is worth to mention that capital-labor ratio for all firms in R&D sector are same. To see this, we just solve firms' profit maximization problem and have

$$\frac{W_t}{N_t} = (1 - \alpha) \eta_t Z \left(\frac{K_t^j}{N_t L_t^j} \right)^\alpha \tag{4}$$

. By using this result and capital return formula above we find that

$$r_t^j = r_t$$

which means every firm has same capital return rate.

After firm j sells its blueprints and get the revenue comes the second stage. Every firm has a probability of π to have investment opportunity and those firms have investment opportunities can transform final product into capital. The technology is 1 unit of final product at period t can be transformed into 1 unit of capital. We assume the market of capital is open after the investment thus firm j has to use the profits it sells the blueprints and external source to invest. We assume the only source of external financing for j is intratemporal loans E_t^j from other firms. Those who borrow from other firms have choice between default or not default. There is no force to ensure borrowers from defaulting so borrowers are required to provide enough collateral to secure their loans. Following Miao and Wang (2018), the value of firm is used as collateral. If the owner of borrower chooses to default and escape with the fund, lenders will take over the firm to compensate their loss. However, we assume the lender may be not familiar with the borrower's firm. There may be a cost during the take over process and the cost is $1 - \xi$ of total capital. Thus the credit constraints are

$$E_t^j \leq \rho_{t+1} V_{t+1}^j (\xi (1 - \delta) K_t^j)$$

Here $V_t \left(K_t^j \right)$ is firm j's cum-dividend equity value when there is capital K_t^j. The credit constraints mean that if borrowers default, the discounted value of the firm left to lenders are no less than the loans so lenders do not have any loss. For borrower j, it is better to

pay back debt E_t^j than default and lose the firm values $\rho_{t+1}V_{t+1}^j((1-\delta)K_t^j)$ so there is no default in this economy.

After investment, all firms come to the third stage at which they can buy and sell capital to each other and pay the dividends. Thus the profit of investment is

$$q_t I_t^j - I_t^j$$

where q_t is the price of capital and I_t^j are how many capital firm j plans to create by investment.

From the setting above, we can write R&D firm j's cum-dividend equity value at period t by using recursive form.

$$
\begin{aligned}
V_t\left(K_t^j\right) &= (1-\pi)\max_{K_{t+1}^j, B_t^j}\left[D_t^j + \rho_{t+1}V_{t+1}\left(K_{t+1}^j\right)\right] \\
&\quad + \pi \max_{K_{It+1}^j, I_t^j, B_t^j}\left[D_{It}^j + \rho_{t+1}V_{t+1}\left(K_{It+1}^j\right)\right]
\end{aligned}
\tag{5}
$$

Here D_t^j and K_{t+1}^j are dividend and capital for next period when there is no investment opportunity while D_{It}^j and K_{It+1}^j are dividend and next period capital when there is investment opportunity. The cum-dividend equity value now is equal to the expected value of dividend plus discounted future cum-dividend equity value when firms make best choice of debt, investment and future capital.

Firms also face some constraints. There are budget constraints (6) and (7)

$$D_t^j + q_t K_{t+1}^j + E_t^j = r_t^j K_t^j + q_t(1-\delta)K_t^j + E_t^j \tag{6}$$

$$D_{It}^j + q_t K_{It+1}^j + E_t^j + I_t^j = r_t^j K_t^j + E_t^j + q_t(1-\delta)K_t^j + q_t I_t^j \tag{7}$$

(6) are budget constraints when there is no investment opportunity while (7) are budget constraints where there is investment opportunity. Investment is constrained by available fund

$$I_t^j \le r_t^j K_t^j + E_t^j \tag{8}$$

At the same time, as we have discussed before debt cannot violate credit constraints otherwise borrowers may default on debts so

$$E_t^j \leq \rho_{t+1} V_{t+1}^j (\xi (1 - \delta) K_t^j) \tag{9}$$

. Bellman equation (5) alone with (3), (6), (7), (8) and (9) consist of R&D firm j's dynamic programming problem.

2.2.5 Competitive Equilibrium

After we describe our model, we can define competitive equilibrium. Let $K_t = \int_0^1 K_t^j dj$, $I_t = \int_0^1 I_t^j dj$, $T_t = \int_0^1 T_t^j dj$ are aggregate capital, investment, new blueprints.

Definition 1 *A competitive equilibrium is defined as allocations*

$\left\{ Y_t, K_t, C_t, I_t, N_t, E_t^j, T_t, L_t^j, I_t^j, K_t^j, T_t^j, Y_t^i, \psi_t^j, X_t^n \right\}$ and prices

$\left\{ w_t, P_n^t, R_t^j, q_t, \eta_t, r_t, V_t^j \right\}$ such that household maximize its utility and firms in all three sectors maximize their profits and market clearing conditions are satisfied which are stock market is clearing $\psi_t^j = 1$, labor market is clearing $\int_0^1 L_t^j dj = 1$, debt market is clearing $\int_0^1 E_t^j dj = 0$, capital market is clearing $K_{t+1} = (1 - \delta) K_t + I_t$, goods market are clearing $C_t + \int_{n=0}^{N_t} \int_0^1 X_t^n dn + I_t = Y_t$ and the amount of patent follows $N_{t+1} = N_t + T_t$.

2.2.6 Analysis of Equilibria

Similar with other endogenous growth model, many variables in our model increase to infinity. Balanced growth path (BGP) is the most important result of these models. To find the BGP, we detrend variables which are increasing with time. Let $c_t = \frac{C_t}{N_t}$, $k_t = \frac{K_t}{N_t}$, $d_t = \frac{D_t}{N_t}$, $t_t = \frac{T_t}{N_t}$, $b_t = \frac{B_t}{N t}$, $w_t = \frac{W_t}{N_t}$, $1 + g_{t+1}^N = \frac{N_{t+1}}{N_t}$. Thus

$$\rho_{t+1} = \beta \frac{c_t}{c_{t+1} \left(1 + g_{t+1}^c \right)} \tag{10}$$

and capital return equation can be written as

$$r_t k_t = Z \eta_t (k_t)^\alpha - w_t \tag{11}$$

We first consider the problem of R&D section. This problem is not a contraction mapping and may have multiple solutions.

Proposition 2 *Suppose $q > 1$, solution of R&D firm j's problem is*

$$V_t\left(K_t^j\right) = a_t K_t^j + B_t \tag{12}$$

where

$$a_t = r_t + q_t\left(1 - \delta\right) + \pi\left(q_t - 1\right)\left(r_t + \rho_{t+1} a_{t+1} \xi\left(1 - \delta\right)\right) \tag{13}$$

$$B_t = \left[1 + \pi\left(q_t - 1\right)\right]\rho_{t+1} B_{t+1} \tag{14}$$

and

$$q_t = \rho_{t+1} a_{t+1} \tag{15}$$

.

Proof. Assume solution of R&D firm j's problem is (12). Substitute (12), (6) and (7) into (5) we have

$$
\begin{aligned}
a_t K_t^j + B_t \;=\; \max_{K_{t+1}^j, K_{It+1}^j, I_t^j, B_t^j} \; & r_t K_t^j + q_t\left(1 - \delta\right) K_t^j \\
& + \rho_{t+1} B_{t+1} + \left(1 - \pi\right)\left[-q_t K_{t+1}^j + \rho_{t+1} a_{t+1} K_{t+1}^j\right] \\
& + \pi\left[\left(q_t - 1\right) I_t^j - q_t K_{It+1}^j + \rho_{t+1} a_{t+1} K_{It+1}^j\right]
\end{aligned}
\tag{16}
$$

and two other constraints (8) and (9) are combined to one constraint

$$I_t^j \leq r_t K_t^j + \rho_{t+1} a_{t+1} \xi\left(1 - \delta\right) K_t^j + \rho_{t+1} E_{t+1} \tag{17}$$

By taking first order derivative of K_{t+1}^j we have (15).

Since $q > 1$, firm j invests as many as it can so (17) is binding. By substituting (17) into (16) and compare the left hand side and right hand side we get (13) and (14). ∎

(15) shows that the price of capital is equal to the value it increases. This is related with Tobin's Q theory. Tobin's Q theory states that if the replacement cost of capital

is less than the firm's value then the firm will increase their investment to have more capital while if the replacement cost of capital is greater than the firm's value then the firm will not invest and decrease capital. Since (15) holds, firm j is indifferent between buying and selling its existing capital. Hence K^j_{It+1} and $K^j_{\bar{I}t+1}$ are indeterminate. We know $q_t \geq 1$ because the marginal cost of producing capital is 1. When $q > 1$, firms with investment opportunities invest as many as they can so (17) is always binding. If $q_t = 1$, however, firms are indifferent in making more investment and credit constraints do not have to bind any more. This is the reason we restrict our main analysis to $q > 1$.

The two equations (13) and (14) play key roles in our model. It shows that R&D firm j's cum-dividend equity value is written as $a_t K^j_t + B_t$. The first term $a_t K^j_t$ means that capital affects the equity value while the second term B_t does not relate with any goods or products. In literatures about bubbles, economists define the first term as fundamental value of a firm while the second term is viewed as bubbles. To see why $a_t K^j_t$ is the fundamental value of the firm, we rewrite (13) with (15) and get

$$q_t = \rho_{t+1} r_{t+1} + \rho_{t+1} q_{t+1} (1 - \delta) + \pi \rho_{t+1} (q_{t+1} - 1) (r_{t+1} + q_{t+1} \xi (1 - \delta)) \qquad (18)$$

If we use $\varphi_{t+1} = \pi (q_{t+1} - 1) (r_{t+1} + q_{t+1} \xi (1 - \delta))$ as the expected investment revenue from next period by increasing one unit of capital, we rewrite it as

$$q_t = \rho_{t+1} \left(r_{t+1} + \varphi_{t+1} \right) + \rho_{t+1} (1 - \delta) q_{t+1} \qquad (19)$$

The solution of (19) is

$$q_t = \sum_{i=t+1}^{\infty} \rho(i, t) (1 - \delta)^{i-t-1} (r_i + \varphi_i) + \frac{\Upsilon}{\rho(t, 0) (1 - \delta)^t}$$

Here Υ is a constant. r_i is the revenue of one unit of capital at period i while φ_i is expected investment revenue for one unit of capital. By using transeversality conditions, $\Upsilon = 0$. Thus $q_t = \sum_{i=t+1}^{\infty} \rho(i, t) (1 - \delta)^{i-t-1} (r_i + \varphi_i)$ is total future income if firm j buy one unit of capital in period t. $a_t K^j_t = \frac{1}{\rho_t} \sum_{i=t}^{\infty} \rho(i, t) (1 - \delta)^{i-t-1} (r_i + \varphi_i) K^j_t$ reflects

the total expected revenue from a firm with K_t^j capital. It is just fundamental value of a firm. It is worth to mention that since our paper uses discrete model rather than continuous time model, a_t is more complicated than Miao and Wang's (2018) model. In their model, $a_t = q_t$ because the return r_{t+1} and depreciation is omitted in continuous time model. The second term of equity B_t, however, is not related with any fundamental future revenue directly and are viewed as bubbles by economists.

Transition of bubbles comes from (14). When there are investment opportunities, bubbles can be used as collateral to increase investment profit by $\pi (q_t - 1) B_t$. Just as the definition given by Miao and Wang (2018), $\pi (q_t - 1)$ is liquidity premium. Later when we discuss the balanced growth path, one can easily show that bubbles grow like this do not violate transversality conditions because the growth is smaller than one when discounted by the discount factor. The reason why people bear the loss to accept such kinds of asset is it can reduce the liquidity mismatch when firms have investment opportunities but are restricted by credit constraints. This is consistent with feature 1 of R&D that rapid technological innovations often correlate with bubbles. When there are investment opportunities, bubbles help firms face credit constraints. Thus, there are more patents created and technological innovations are faster. This effect is similar with crowd in effect in most literature about bubbles. It is deserved to mention that this is just the direct effect of bubbles. When we compare balanced growth rate between bubbly equilibrium and bubbleless equilibrium we will find there are also indirect effects of bubbles and they may offset some of crowd in effect.

Another observation of (14) is bubbles either exist from the beginning or they never appear. As we have discussed before, the dynamic programming problem is not a contraction mapping and may have multiple solutions. We have two cases here, an equilibrium with bubbles and an equilibrium without bubble. The existence of bubbles is just a consensus of the market not relating with any fundamental of a certain firm. If all agree and believe other will accept the extra values then the bubbles exist. We detrend (14) into

$$b_t = [1 + \pi (q_t - 1)] \rho_{t+1} (1 + g_{t+1}) b_{t+1} \tag{20}$$

Before we move on to study two different equilibria, we first discuss a little further to the general case. We have already known every inventor has the same capital labor ratio. Thus

$$T_t = \int_0^1 T_t^j dj = Z \left(K_t\right)^\alpha N_t^{1-\alpha}$$

$$N_{t+1} = N_t + Z \left(K_t\right)^\alpha N_t^{1-\alpha}$$

$$1 + g_{t+1}^N = 1 + Z k_t^\alpha$$

$$g_{t+1}^N = t_t \tag{21}$$

$$t_t = Z k_t^\alpha \tag{22}$$

$$(1 + g_{t+1}^N)k_{t+1} = (1 - \delta) k_t + \pi r_t k_t + \pi \rho_{t+1} \left[a_{t+1} \left(\xi \left(1 - \delta\right)\right) k_t + (1 + g_{t+1}^N)b_{t+1} \right] \tag{23}$$

$$r_t = \alpha \left(Z\eta_t\right)^{\frac{1}{\alpha}} \left(\frac{w_t}{1 - \alpha}\right)^{\frac{\alpha-1}{\alpha}} \tag{24}$$

$$c_t + X_t + \pi r_t k_t + \pi \rho_{t+1} \left[a_{t+1} \left(\xi \left(1 - \delta\right)\right) k_t + (1 + g_{t+1}^N)b_{t+1} \right] = AX_t^\sigma \tag{25}$$

Thus, equations (1), (2), (10), (11), (13), (15), (20), (21), (22), (23), (24) and (25) alone with transversality conditions consist of a dynamic system which characterize the detrended equilibria of our model.

2.3 Balanced Growth Path

In this section, we derive and compare balanced growth path (BGP) of two cases. The first one is the case when there is no bubble while the second one is the case with bubbles. We have detrended all variables in last sector so variables should be at steady state alone BGP. We use detrended variables without time subscript to denote the steady state of these variables. Alone BGP, we know that $g^N = g^C$. We will use g^N as a substitute when there is g^C for convenience. We first show that in both cases, given q and r, capital k and growth rate g^N are determined in same way.

Proposition 3 *k is determined implicitly by equation*

$$rk = Z\frac{\left(\frac{1-\sigma}{\sigma}\right)\sigma^{\frac{2}{1-\sigma}}A^{\frac{1}{1-\sigma}}}{1-\beta\left(1+Zk^\alpha\right)^{-1}}k^\alpha - (1-\alpha)\,r^{\frac{\alpha}{\alpha-1}}\alpha^{\frac{\alpha}{1-\alpha}}\left[Z\frac{\left(\frac{1-\sigma}{\sigma}\right)\sigma^{\frac{2}{1-\sigma}}A^{\frac{1}{1-\sigma}}}{1-\beta\left(1+Zk^\alpha\right)^{-1}}\right]^{\frac{1}{1-\alpha}} \tag{26}$$

and

$$g^N = Z\left(k\right)^\alpha \tag{27}$$

Proof. (27) is the direct result of (21) and (22). We only need to get (26) then growth rate is determined by q, a and r. From (10),

$$\rho = \beta\frac{1}{(1+g^N)} = \beta\left(1+Zk^\alpha\right)^{-1}$$

This result alone with (2) give

$$\eta = \frac{\left(\frac{1-\sigma}{\sigma}\right)\sigma^{\frac{2}{1-\sigma}}A^{\frac{1}{1-\sigma}}}{1-\rho} = \frac{\left(\frac{1-\sigma}{\sigma}\right)\sigma^{\frac{2}{1-\sigma}}A^{\frac{1}{1-\sigma}}}{1-\beta\left(1+Zk^\alpha\right)^{-1}}$$

From (24) we have

$$w = (1-\alpha)\,r^{\frac{\alpha}{\alpha-1}}\alpha^{\frac{\alpha}{1-\alpha}}\left[Z\frac{\left(\frac{1-\sigma}{\sigma}\right)\sigma^{\frac{2}{1-\sigma}}A^{\frac{1}{1-\sigma}}}{1-\beta\left(1+Zk^\alpha\right)^{-1}}\right]^{\frac{1}{1-\alpha}}$$

Substitute these results into (11) we get (26). ■

2.3.1 Bubbleless BGP

In bubbleless equilibrium we know $b_t = 0$. From (20) we know that if there is no bubble in one period, there is no bubble for all periods. Thus equation (20) becomes an identity. At the same time, (23) becomes

$$(1+g^N_{t+1})k_{t+1} = (1-\delta)\,k_t + \pi r_t k_t + \pi\rho_{t+1}\left[a_{t+1}\left(\xi\left(1-\delta\right)\right)k_t\right] \tag{28}$$

Proposition 4 *When credit constraints are binding, capital price q_l capital return r_l, detrended capital k_l and growth rate g_l^N is determined by*

$$1 + g^N = (1 - \delta) + \pi r + \pi q \left(\xi \left(1 - \delta \right) \right) \tag{29}$$

$$\frac{q(1 + g^N)}{\beta} = r + q \left(1 - \delta \right) + \pi \left(q - 1 \right) \left(r + q \xi \left(1 - \delta \right) \right) \tag{30}$$

alone with (26) and (27).

Proof. We get (29) by substituting (15) into (28). By (15) and (13) we have (30). (29), (30) alone with (26) and (27) we derive from proposition 2, we get a four variables equations system which give us q_l, r_l, k_l and g_l^N. ∎

Unfortunately, it is impossible to derive the analytical solution of the variables. Thus, later we cannot compare the results between bubbleless BGP and bubbly BGP directly. However, we can use numerical method to check the results.

2.3.2 Bubbly BGP

We now study bubbly BGP. Here $b_t \neq 0$ and we cannot omit the (20). Just like what we have discussed in Bubbleless BGP, next proposition gives us the result of capital price q capital return r, detrended capital k and growth rate g^N. We use q^b, r^b, k^b and g_b^N as denotation.

Proposition 5 *When there are credit constraints,*

$$q^b = \frac{1 - \beta + \pi\beta}{\pi\beta}$$

r^{bb}, k^{bb} and g_{bb}^N are determined by

$$
\begin{aligned}
\frac{q(1 + g^N)\left(1 - \beta + \pi\beta \right)}{\pi\beta^2} = {} & r + \frac{\left(1 - \beta + \pi\beta \right) \left(1 - \delta \right)}{\pi\beta} \\
& + \frac{1 - \beta}{\beta} \left(r + \frac{1 - \beta + \pi\beta}{\pi\beta} \xi \left(1 - \delta \right) \right)
\end{aligned}
\tag{31}
$$

alone with (26) and (27).

Proof. Consider the case alone BGP. (20) gives us q^b. By (15) and (13) we have

$$\frac{q(1+g^N)}{\beta} = r + q\left(1-\delta\right) + \pi\left(q-1\right)\left(r + q\xi\left(1-\delta\right)\right)$$

. Plug q^b into it we have (31). (31), (26) and (27) are a three variables three equations system. ■

It is easy to check that $q^b > 1$ which means when collateral constraints are binding, bubbles always exist and the existence of bubbles never totally eliminate the effects of collateral constraints.

2.3.3 Compare Bubbly BGP with Bubbleless BGP

According to proposition 2, 3 and 4, we know how to get values of variables on BGP. Unfortunately, it is impossible to get most results analytically for the system is too complicated. We use numerical method alone with equations we derive before to discuss how bubbles affect R&D and how credit constraints affect bubbles. The parameters are reported in table 1.

Table 1: Values of Parameters

parameter	β	δ	π	ξ	α	σ	A	Z
value	0.95	0.15	0.04	0.5	0.3	0.99	1	0.42

We can get both bubbly BGP and bubbleless BGP. Some important variables alone the BGP are reported below in table 2. Here e is detrended debt. Compared with BGP

Table 2: Variables' values alone bubbly and bubbleless BGPs

variable	$b/v(k)$	g^N	q	a	r	k	e	$v(k)$
bubbly	57.59%	4.0%	2.32	2.53	0.49	0.00038	0.0016	0.0023
bubbleless	0	3.5%	9.26	10.09	0.69	0.00025	0.0010	0.0025

without bubble, bubbly BGP has higher detrended capital level. Thus, the growth rate of bubbly BGP is greater. At the same time, capital price q, relationship between capital and equity value a and capital return rate r decrease. The detrended stock value of bubbly BGP, however, is less than the bubbleless one. To understand these phenomena, we first review the detrended capital transition (23). From transition function, we know

that capital increases if revenue increases which means $\pi r_t k_t$ increases or firms have more access to external funding which means $\rho_{t+1} \left[a_{t+1} \left(\xi \left(1 - \delta \right) \right) k_t + \left(1 + g_{t+1}^N \right) b \right]_{t+1}$ increases.

Bubbles increase discounted stock price if lenders take over the firms thus provide more collateral to help reduce liquidity mismatch. This direct effect increases investment in R&D sector so it is the positive effect on growth rate. This positive effect can be called crowd in effect like other literatures (Hirano and Yanagawa 2017, Miao and Wang 2018) about bubbles. If we only consider this direct effect, bubbles certainly help R&D and growth.

However, there is also some indirect effects in general equilibrium which offset some of the positive effects. First of all, capital in our model not only be used to create new patents, but also be used to increase cum-dividend equity value so that when they have investment opportunities they can borrow more. Since bubbles in stock price have the same effect, demand of capital decreases which decreases price of capital q. That's why we see in both examples q and a drop significantly when there are bubbles. This negative effect offsets some positive effects especially when credit constraints are not binding very much.

It is worth mention that this direct effect does not ensure higher detrended stock price of bubbly BGP. This is because indirect effects reduce the stock price by reducing a at the same time bubbles increase the stock price. Sometimes the indirect effects are not too big so the stock price still increases while sometimes the indirect effects are big enough so the stock price may decrease. However, collateral constraints are related with stock price if lenders take over and this stock price always increases. This is the just we show in figure 1 in introduction.

That is not the end of the story. The effects we discuss above only ensure alone bubbly BGP firms get more loans. Since capital price q decreases, return of capital r also decreases which means that even with same amount of capital firm get less revenue through innovation activities and has to decrease the investment. Though there are these two negative effects which offset some of the positive effect, the positive effect is always the dominant one so stock price bubbles always encourage investment in innovation sector

and increase growth rate. We can see this result when we do robust check. It is worth to mention that these direct and indirect effects are very similar with Oliver's finding.

2.3.4 Robustness

In some literature about bubbles, bubbles may only exist in some economies satisfy some certain conditions. Hirano and Yanagawa (2017) find that bubbles only exist when an economy had intermediate financial frictions in their model. Thus, this kinds of bubble region restrict the usefulness of the model. Although it is impossible to derive the conditions under which bubbles exist analytically, we can use numerical way to show that bubbly BGP is rather robust so under most situations bubbles may exist. We also show that the result bubbly BGPs always have higher growth rate is robust. What is more, the tighter credit constraints are, the more benefit bubbles bring.

Our study focuses on two parameters ξ and π. ξ reflects the taking over cost when default. The smaller the ξ is, the more costly the taking over is. Thus, credit constraints are tighter. During the robust test, we assume other parameters will have values the same as the study in previous subsection.

Table 3: Robustness of credit constraints

ξ	1	0.7	0.3	0.1
$b/V(k)$	49.54%	54.69%	60.15%	62.39%
g	4.1%	4.0%	3.9%	3.8%

Table 3 is the result showing how different ξ affect bubbles. $b/V(k)$ characterize the average size of bubbles compare with firms' value. When ξ decreases, credit constraints are tighter and tighter. Growth rate decreases and bubbles increase. Even when $\xi = 0.1$ which means lenders can only take over 10% of original capital when borrowers default, bubbles still exist. This test means our model is very robust on financial conditions. Bubbles is possible to exist even in an economy with extremely tight credit constraints. Besides ξ, π is another parameter we have interested in. π is the probability a firm find an investment opportunity in one period. If the probability is higher, more firms have investment opportunities thus the economy is more efficient in reallocating social resource. Thus the credit constraints are not binding so tight as before. For this reason,

bubbles are shrinking quickly with the increasing of π. This can be seen in numerical analysis.

Table 4: Robustness of opportunities

π	0.003	0.04	0.1	0.2
$b/V(k)$	70.37%	57.59%	37.94%	11.89%
g	1.9%	4.6%	4.7%	4.9%

What is the relationship when both ξ and π change? We give a region in which bubbly BGP are possible to exist. As usual, other parameters are the same as before. As shown in figure 3, the shade area is the region where bubble are possible to exist.
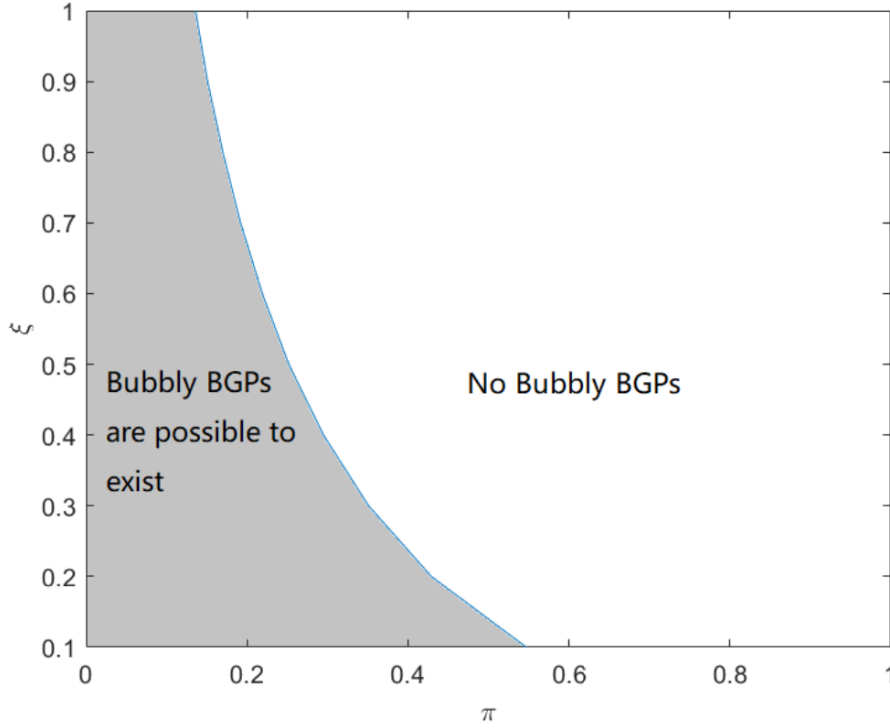

Figure 3: Region in which bubbly BGPs are possible to exist.

With the decreasing of ξ, bubbles may exist in economy with higher chance of investment opportunities. This is because decreasing of ξ makes credit constraints tighter. Even there are a large number of firms can invest, they still want to borrow more. From figure 3, we find that bubbles are possible to exist in many different cases which means our model is more useful than Hirano and Yanagawa (2017). Figure 4 gives us how π and ξ affect bubbles. Generally speaking, if π decreases of ξ decreases, credit constraints are tighter, thus value of bubbles are greater.
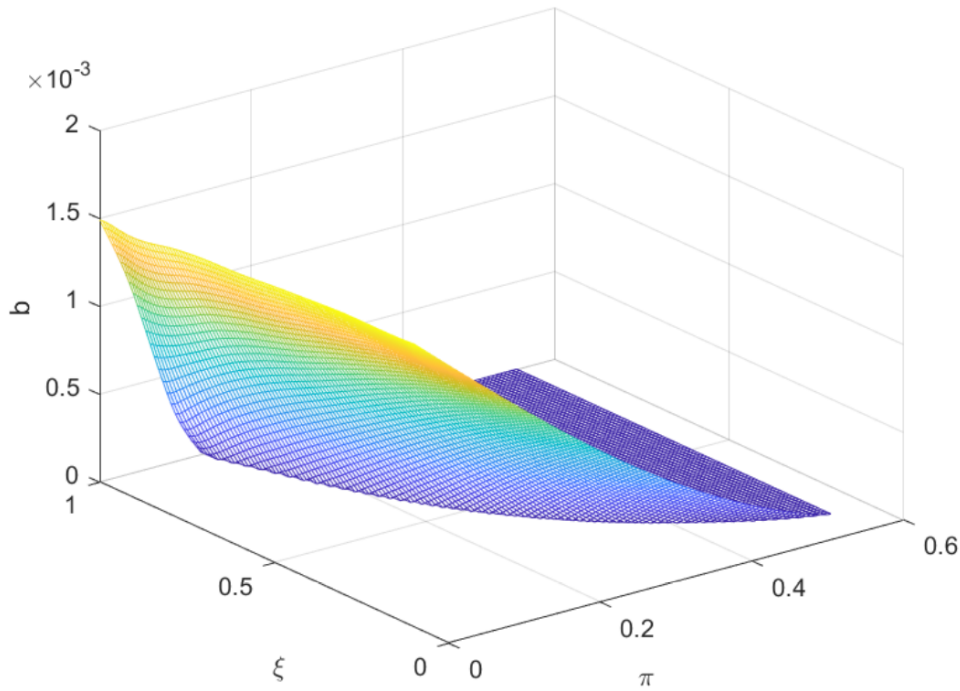
Figure 4: Bubbles with different π and ξ

Figure 5 gives us the information on the difference of growth rate between bubbly BGPs and bubbleless BGPs. Bubbly BGPs always have higher growth rate. At the same time, the smaller the π and ξ are, the bigger the difference it is. The reason is very simple, smaller π and ξ increase the tightness of credit constraints so bubbles play more important role in the economy.

2.4 Dynamics

In this section, we study the dynamics of the model. We first study the transition around bubbly BGP. We then study what happens when bubbles burst unanticipatedly. The stochastic burst of bubbles will be studied in next section where we extend our baseline model into a stochastic model.

2.4.1 The Dynamics around Bubbly BGP

Since we have a big dynamic system, we are unable to derive analytical results for local dynamics. However, we can solve it numerically. We find rank conditions are satisfied
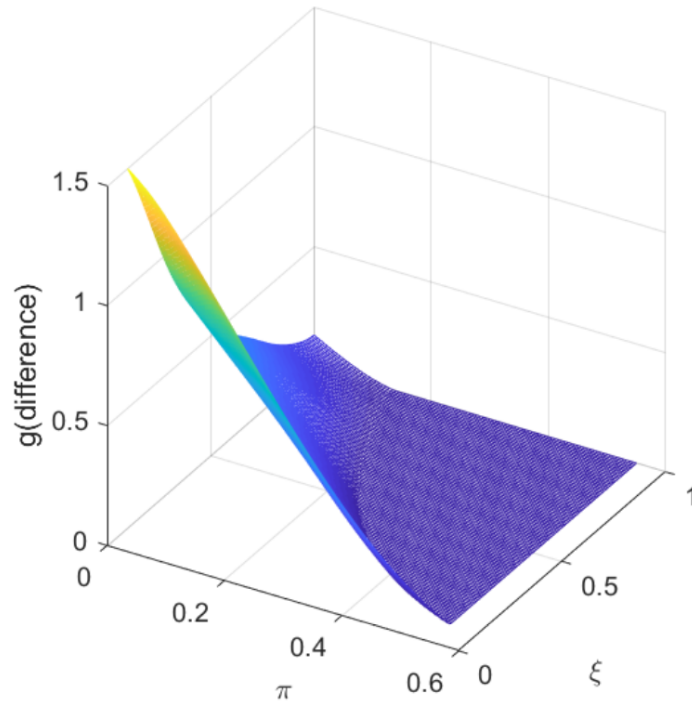
Figure 5: Difference of growth rate between bubbly and bubbleless BGPs

for our examples. As in figure 6, we start from the point where detrended capital is about 10% more than alone BGP path. At this point, detrended bubbles is about 2.8% smaller than the BGP bubbles. With time going on, detrended capital is decreasing while detrended bubbles are increasing. In the end, they converge to the level of those alone BGP with bubbles.

2.4.2 Unanticipated Burst of Bubbles

One of the most obvious feature of bubbles are bubbles tend to burst. For example, when Dot-com bubbles burst suddenly, Nasdaq Composite index fell 25% in one week. The price of Bitcoin dropped from around $19000 to around $6000 in less than two months. (He 2018) Very few people realize the bubbles is going to burst before it really happens. There are two ways to deal with burst of bubbles. The first approach is bubbles will burst unanticipatedly. However, many economists believe that although people do not know when bubbles burst, they expect bubbles will burst sooner or later. Thus the second approach is stochastic bubble. We study how unanticipated burst of bubbles have
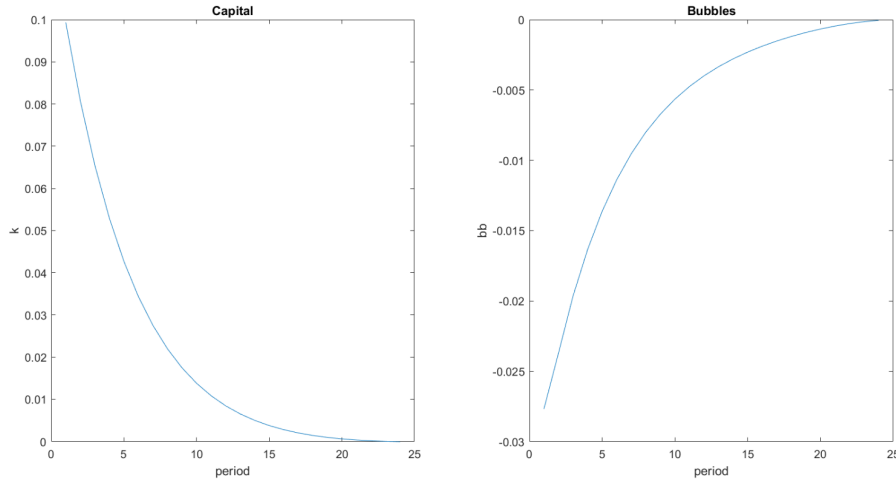
Figure 6: Dynamics when there are 10% more capital than alone bubbly BGP

effects on the economy in our model. In next section, we study what happens when bubbles burst stochastically.

We assume the economy is growing alone bubbly BGP when there is an unanticipated shock at period 2. The shock changes the consensus that people believe others will not accept the overvaluation of equity. Thus, bubbles burst and there is no bubble from that periods on. Figure 7 is the result of what happens when bubbles burst.

From figure 7 we can see what happens with other variables when bubbles burst. Since there is no bubble any more, credit constraints bind tighter and firms cannot get so much loans as before. Firms have to reduce their investment which leads to the decreasing of detrended capital from period 3. Capital gradually converges to the level of bubbleless BGP. Growth is also slower because innovation is slowed with the limit of capital. Capital price, however, jump at the time of bubbles burst and then grows slowly. The jump of price is due to the jump of capital demand since capital is now the only instruments to be used to increase the value of collateral. After that, amount of capital is decreasing which leads to the scarcity of capital which drives the price up gradually and also increases capital return rate. Return of capital r increases. When bubbles burst, the jump of capital price increases capital return rate immediately. After that, capital return rate goes up gradually with the increasing of capital and capital price. a increases
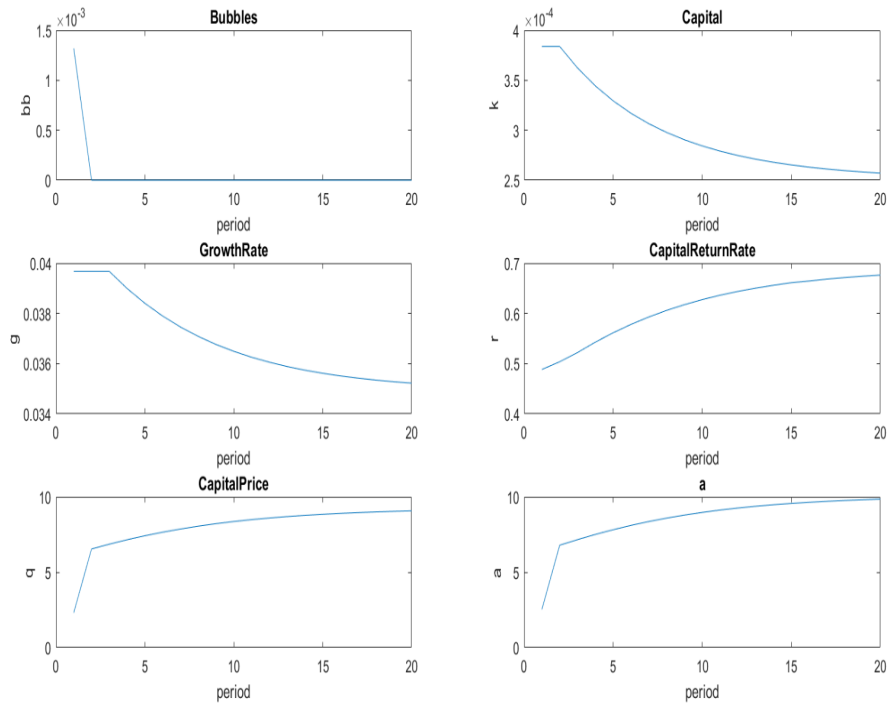
Figure 7: Unanticipated burst of bubbles

following the same pattern of capital price. Thus, economy will converge to bubbleless
BGP gradually if bubbles burst unanticipatedly.

2.5 Extensions

In this section, we study two extensions of our model. In the first extension, we
introduce stochastic bubbles into our model. Up to now, bubbles are deterministic and
they only burst when there is an unanticipated shock. People do not believe bubbles may
burst unless it really happens even though they admit there are bubbles in stock market.
These assumptions are not very realistic. Though it is hard to know when bubbles burst,
people believe bubbles will burst in the future and the probability of burst affects people's
decision. To reflect this, we assume bubbles may burst every periods with a probability.

The second extension is studying how bubbles affect resource allocation. In the base-
line model, we focus on how bubbles affect R&D sector directly. However, bubbles in one
sector may have reallocation effects of resource. In the second extension, we assume both

final product sector and R&D sector must use labor to produce and labor flow from one sector to the other sector freely. Thus, bubbles in R&D sectors can reallocate labor.

2.5.1 Stochastic Bubbles

We assume if there are bubbles, they may burst at probability θ every period. The burst will happen before R&D activities. Other setting are the same as baseline model. If bubbles burst, everything works like the bubbleless equilibrium in the baseline model. We only need to consider the case when there are bubbles. Everything is the same as baseline model except (5) becomes

$$
\begin{aligned}
V_t\left(K_t^j\right) &= (1-\pi)(1-\theta) \max_{K_{t+1}^j, B_t^j}\left[D_t^j + \rho_{t+1}V_{t+1}\left(K_{t+1}^j\right)\right] \qquad (32)\\
&\quad + \pi(1-\theta) \max_{K_{It+1}^j, I_t^j, B_t^j}\left[D_{It}^j + \rho_{t+1}V_{t+1}\left(K_{It+1}^j\right)\right] + \theta V_t^{\#}\left(K_t^j\right)
\end{aligned}
$$

where $V_t^{\#}\left(K_t^j\right)$ is the bubbleless cum-dividend equity value we have derived before. We show the next proposition in Appendix

Proposition 6 *When there are bubbles,* $V_t\left(K_t^j\right) = a_t K_t^j + B_t$ *where*

$$
a_t = (1-\theta)r_t + (1-\theta)q_t(1-\delta) + (1-\theta)\pi(q_t-1)\left[r_t + \rho_{t+1}a_{t+1}\xi(1-\delta)\right] + \theta a_t^{\#} \quad (33)
$$

$$
B_t = (1-\theta)\left[1 + \pi(q_t-1)\right]\rho_{t+1}B_{t+1} \qquad (34)
$$

We can see the stochastic bubbles model has the similar result with baseline model. The only difference between (33), (34) and (13), (14) are the probability of burst. There is $1-\theta$ chance that bubbles still exist at the beginning of that period thus we have terms similar like before. However, there is probability θ that bubbles burst. If bubbles do burst, firms will operate as the firms in bubbleless equilibrium so we have the term $\theta a_t^{\#}$. We then use numerical method to study the stochastic model.

Here we have $\theta = 0.05$ and all other parameters are same as the baseline model. The results are in figure 8. After the burst of bubbles, the path to the bubbleless equilibrium is similar with the unanticipated shock. The mechanism is the same as baseline model.
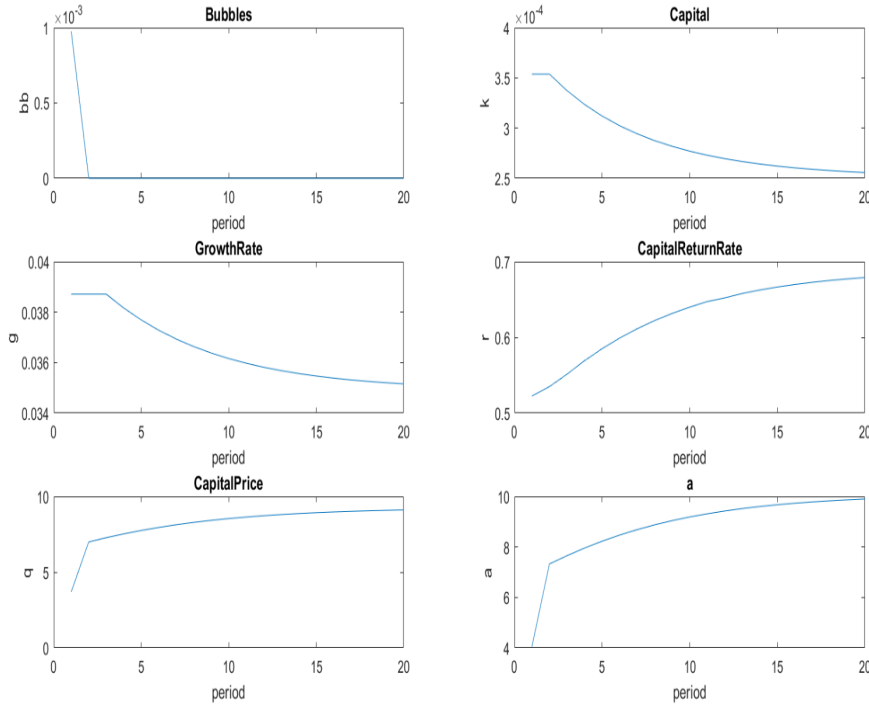
Figure 8: Burst of stochastic bubbles

2.5.2 Reallocation Effects Model

We assume the household has labor supply $\bar{L}$. Budget constraint of the representative household is $C_t + \int \left(V_t^j - D_t^j\right)\psi_{t+1}^j dj = \int V_t^j \psi_t^j dj + W_t \bar{L}$ Final goods producer now has the technology $Y_t = A\int_{n=1}^{N_t}\left(X_t^n\right)^\sigma \left(L_t^Y\right)^{1-\sigma}dn, 0 < \sigma < 1$. Here L_t^Y is the labor hired by final goods producer. Thus, only $\bar{L} - L_t^Y$ labor works in R&D sector. Since most derivation and results are similar with baseline model. We put all the derivation into appendix. We only provides the numerical results here.

Given the parameters in baseline model and $\bar{L} = 2$, we have bubbly BGP and bubbleless BGP in table 5. The results are similar like we have discussed in baseline model.

Table 5: Variables alone BGPs in reallocation effects model

	$b/V(k)$	g^N	q	a	r	k	L^Y
bubbly	53.97	1.7%	2.32	2.48	0.44	0.00010	1.46
bubbleless	0	1.4%	8.22	8.77	0.60	0.00006	1.50

The only difference is that labor works in final products sector in bubbly economy is less than the labor in bubbleless economy. Which means labor flow from final products sector to R&D sector. The intuition is simple. We have discussed that there are more capital

in R&D sector when there are bubbles thus the marginal productivity of labor in R&D sector is higher. R&D firms would like to hire more labor to produce new blueprints. This effect alone with the effects we discussed before increases the positive effects of bubbles in R&D sector and helps the economic growth.

We also study the burst of bubbles. As the baseline model, the pattern of burst both in unanticipated shock and stochastic bubbles show the similar result. We will only report the unanticipated shock here in figure 9. The stochastic bubbles are reported in appendix.

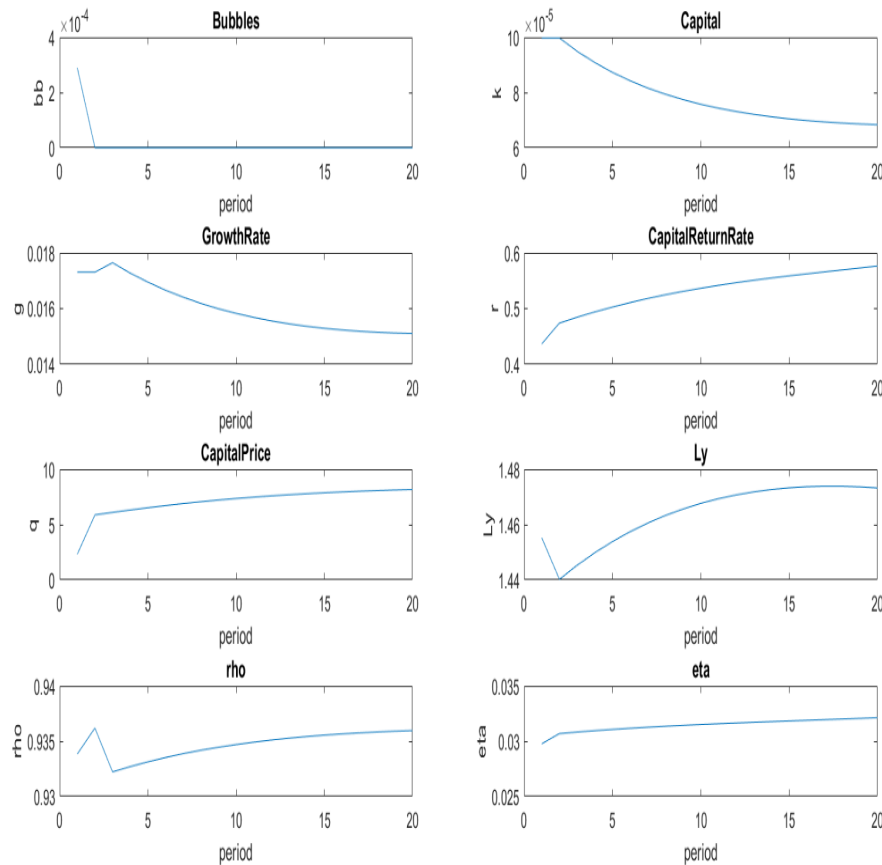Figure 9: Unanticipated burst of bubbles in reallocation effects model

Most of the path are similar with baseline model. The only significant difference is when bubbles burst, growth rate first increases and then decreases gradually in this model while growth rate decreases with the decreasing capital in baseline model. This may be a surprising result. Why does the economy grow faster when the bubbles burst and

capital start decreasing? This is because the change of stochastic discount rate ρ_t. Since ρ_t increases generally after the burst of bubbles, the price of patent η_t also increases. Although capital decreases soon after the burst of bubbles, increasing of patent price increases the marginal productivity of labor in R&D sector thus it attracts more labor flows out of final goods sector and works for R&D sectors. This effect compensate the decreasing of capital. At the beginning, this reallocation effect is strong enough so there are more patents are produced. With less and less capital, marginal productivity of labor is decreasing and people flow out of R&D sector and the growth rate is smaller and smaller until it reaches the bubbleless BGP.

2.6 Conclusion

In this paper, we introduce credit constraints into a standard endogenous growth model with innovation. We show that there are multiple equilibria and stock price bubbles exist in one of the equilibria. To the extent of our knowledge, this is the only study about endogenous growth with bubbles by using a well defined R&D growth model and it is the only model in which bubbles are generated by features of innovation sectors. Our study finds that stock price bubbles in innovation sector encourage innovation and increase growth rate by reducing liquidity mismatch. Economy with tighter credit constraints benefit more from bubbles. Thus, it may be wise for governments not to make bubbles in innovation sector burst but use policy instruments to reduce financial frictions in innovation sector. This paper can be a bridge between traditional growth model and bubbles.

Besides the economic phenomenon we discussed in this paper. There are some extension which can be done in the future. Our paper has R&D sector and bubbles. At the same time, household buy shares of R&D firms. These are close to the situation the venture capitalists face. In the future, we can extend the R&D sector and this model may be used to study venture capitalists' behavior.

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CHAPTER 3. COLLATERAL CONSTRAINTS, FINANCIAL CRISIS AND OPTIMAL MONETARY POLICY[1]

3.1 Introduction

We study optimal monetary policy in the presence of collateral constraints. For this purpose, we introduce cash-in-advance (CIA) constraints into a version of the Kiyotaki and Moore (1997) economy, one devised by Cordoba and Ripoll (2004b). The model exhibits simple tractable heterogeneity by considering two separate groups of individuals who differ in their rates of time preference. In equilibrium, the more patient group become lenders while the impatient group become borrowers. We take model seriously and exploit its tractability to derive some sharp results regarding optimal monetary policies and optimal monetary responses to financial and non-financial shocks. The government controls the money supply via helicopter drops. The simplicity of the structure allows us to clearly describe the mechanisms at work. The main contribution of the paper is to characterize optimal monetary policy, in the Ramsey sense, both in the short run and the long run. Our paper is also close to Cordoba and Ripoll (2004a), who also introduce cash via CIA constraints but in a more stylized version and without studying optimal policies.

We find that the optimal monetary policy eliminates the social cost of the collateral constraints in the long run. The results is novel. Absent credit constraints, money is superneutral and optimal monetary is undetermined in the steady state. In contrast, money is neutral but not superneutral when collateral constraints are binding. In particular, the price of collateral is a negative function of the inflation rate due to the fact that cash is needed for capital purchases, the collateral in the economy. By lowering the inflation rate, the planner increases the price of capital and relaxes credit limits. The optimal monetary policy is a generalized version of the Friedman rule, one that takes into account the degree of credit tightness. The optimal inflation rate is a positive function of the degree of credit tightness, the cash-component of investment, and of borrowers'

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[1]This is a joint work with Juan Carlos Cordoba

discount rate. The Friedman rule, one that drives the nominal interest rate to zero, is obtained as an special case when the full value of the capital can be collateralized, and borrowers share the same discount rate of lenders. We also find that monetary policy does not work very well in reducing welfare losses of business cycles caused by TFP shocks but it is very effective in reducing welfare losses caused by financial shocks.

A large body of research utilizes Kiyotaki and Moore's mechanism, and part of this literature includes a monetary component. Cordoba and Ripoll (2004a) study the role of collateral constraints in transforming small monetary shocks into large persistent output fluctuations by using CIA to introduce money. Iacoviello (2005) studies a monetary new Keynesian model by using house as collateral. Monacelli (2009) uses a two sector New Keynesian model with durable goods and nondurable goods. Araujo, Schommer and Woodford (2015) consider an unconventional policy which central bank can purchase risky asset. Cao (2017) study the Ramsey optimal fiscal and monetary policy to fiscal shocks. Her paper uses some similar technique as our paper. However, her paper does not provide a long run rule for monetary policy and does not answer if monetary policy itself can be used to deal with business cycles. Liu, Wang and Zha (2013) study changing price of collateral land's effect on macroeconomic fluctuations and Jermann and Quadrini (2012) find financial shocks such as change of collateral contributes more to macroeconomic fluctuations. However, they do not introduce money into their models nor discuss the long term effect of financial frictions in detail.

A number of papers study monetary policy in the presence of credit frictions. Fiore and Tris (2012) and Curdia and Woodford (2017) are recent examples. They both use new Keynesian models with credit spreads to introduce a form of market imperfections. However, as with most other traditional new Keynesian models, monetary policy only works through the interest rate. They ignore other channels of monetary policy and money itself is dispensable. The optimal monetary policy at steady state makes the inflation rate zero because the only long term role of monetary policy is to eliminate the inefficiency associated to price stickiness. Also, many people are facing with a situation that even they would like to borrow at given interest rate they cannot get what they

want to borrow when financial crisis happens. Although credit spread is a good way to introduce financial incompleteness, it cannot reflect this scenario.

Our paper is also related to the literature on the distributive effects of monetary policy, in particular optimal policies, which has attract economists' attention in recent years. A classical paper in this topic is Doepke and Schneider (2006). Doepke and Schneider empirical study to find the redistribution channel of monetary policy. Auclert (2017) builds a model to reflect three channels redistribution effect. Our model, though simple, provides a new channel of redistribution. We find that when there is a financial friction such as collateral constraints and cash plays some role in an economy, an inflation changes the cost of buying capital and in the end lead to the reallocation of capital. It shows that redistribution effect of monetary policy itself can have significant affect on the economy when there is incompleteness in financial market even without any assumptions in new Keynesian setting like most other monetary policy literature has done. It is because incompleteness of financial market makes social resource cannot be used optimally. Monetary policy can redistribute social resources and reduce the welfare loss. For this reason, there must be a optimal monetary policy in the long run which depends on the incompleteness of financial market. In short run, it can reduce the welfare loss generated by financial shock but cannot help TFP shock because the latter has lump-sum effect rather than changes the distribution of resources.

The paper is organized as follow. Section 2 describes the model and derives the conditions of competitive equilibrium. Section 3 characterizes the competitive equilibrium both in the long run and short run. The role of monetary policy and the effects of shocks are also studied in this section. Section 4 studies the Ramsey problem and derives optimal monetary policies.

3.2 The Model

3.2.1 Individuals

There are three types of goods in the economy: consumption, c, capital, k, and money, m. There are two types of individuals, type 1 and type 2, with population sizes of measure

n and 1 respectively. They both have standard preferences. Their utility functions take the form

$$\sum_{t=0}^{\infty} \beta_i^t u(c_{it}), \ i \in \{1, 2\}, \tag{1}$$

where β_i is a discount factor, c_{it} is consumption of agent i in period t, and u is a standard utility function. The following is the key assumption of the model.

Assumption 1. $1 > \beta_1 > \beta_2 > 0$.

According to this Assumption 1, type 1 agents are more patient than type 2 agents. As in Kiyotaki and Moore (1997) or Cordoba and Ripoll (2004a and 2004b), this assumption provides a convenient way to introduce tractable heterogeneity, with type 1 agents eventually becoming the sole lenders while type 2 agents eventually becoming the sole borrowers. More importantly, the different discount factors make the collateral constraint bind and generates an inefficient allocation of resources, as explained below. The assumption adds importance for our goal, of characterizing optimal policies, because it also creates a difference between the social and private rate of discounts. Such difference is at the core of the optimal monetary policy derived in Section 4. We also assume that there is no uncertainty in the model except for the unanticipated shock.

Individuals utilize their capital and one unit of labor to produce y_t units of output at time t. In particular $y_t = e^{A_t} f(k_{t-1})$ where e^{A_t} is the productivity of period t and $f(.)$ is a standard strictly concave function. An agent's total resources in period t includes production, y_t, real loans, b_t, real money holdings, m_{t-1}, and government transfers, t_t. Resource are used to consume, buy capital, accumulate money, and repay loans. The budget constraint of type i agent is

$$q_t(k_{i,t} - k_{i,t-1}) + c_{i,t} + m_{i,t} + (1 + r_t)b_{i,t-1} = y_{i,t} + b_{i,t} + \frac{m_{i,t-1}}{(1 + \pi_t)} + \frac{t_t}{p_t} \tag{2}$$

where p_t is output price, R_t is the nominal interest rate, q_t is real price of capital, and π_t is the inflation rate defined as $\pi_t := \frac{p_t}{p_{t-1}} - 1$. The real interest rate, r_t, is defined as $1 + r_t = R_t / (1 + \pi_t)$.

Agents face Cash-in-Advance (CIA) constraints for consumption and investment as described by

$$\eta q_t(k_{i,t} - k_{i,t-1}) + \zeta c_{i,t} = \frac{m_{i,t-1}}{(1 + \pi_t)} + \frac{t_t}{p_t}. \tag{3}$$

Specifically, a proportion of η investment and a proportion of ζ consumption needs to be paid in cash.

To prevent default, borrowers are required to provide enough collateral to secure their loans. Capital serves as collateral but only up to a fraction γ_t is recovered upon default. The amount of borrowing is thus limited to $b_{i,t} \leq \gamma_t q_{t+1}^n k_{i,t}/R_{t+1}$ in nominal terms, or

$$(1 + r_{t+1})b_{i,t} \leq \gamma_t q_{t+1} k_{i,t} \tag{4}$$

in real terms. The exogenous variable γ_t captures the degree of financial development, and we will use this variable to study exogenous financial shocks.

3.2.2 Government

The government is just a central bank fully in control of the money supply, M_t^s. The government prints money in the amount $M_t^s - M_{t-1}^s$ at period t, which is then distributed equally among the population. Specifically,

$$t_t = \frac{M_t^s - M_{t-1}^s}{1 + n}.$$

In this section money supply is exogenous. Next section studies optimal monetary policy.

3.2.3 Resource Constraints

There are four economywide resource constraints in the economy: money, capital, assets and consumption goods. We assume that the aggregate amount of capital is fixed at the amount K, as in Kiyotaki and Moore (1997).

Since we have the setting like this, the following are the corresponding resource constraints:

$$\frac{M_t^s}{p_t} \geq nm_{1,t} + m_{2,t}$$

$$\overline{K} \geq nk_{1,t} + k_{2,t} \tag{5}$$

$$nb_{1,t} + b_{2,t} \geq 0 \tag{6}$$

$$ny_{1,t} + y_{2,t} \geq nc_{1,t} + c_{2,t}$$

3.3 Competitive Equilibrium

This section defines and characterizes the competitive equilibrium. A first subsection describes optimal conditions and derives some implications. A second subsection focuses on steady state properties of the competitive equilibrium. We show that, under minimal restrictions, a unique steady state solution exists and characterize the "long run" effects of monetary policy and financial development, as described by γ. A third subsection uses numerical simulations around the steady state to characterize the impulse response functions of three types of shocks: productivity, A_t, financial, γ_t, and monetary. The simulations will help us understand the role of optimal monetary policy derived in a later section.

Definition 1 *A Competitive Equilibrium are sequence of prices, $\{p_t, R_t, q_t\}_{t=0}^{\infty}$, and allocations, $\{c_{i,t}, m_{i,t}, k_{i,t}, b_{i,t}, t_t\}_{t=0}^{\infty}$, such that: (i) households utility (1) is maximized subject to (2), (3), (4) given prices; markets for money, capital and loans all clear given exogenous sequences $\{A_t, \gamma_t, M_t^s\}_{t=0}^{\infty}$.*

Goods market clearing is not explicitly considered in this definition as it automatically clears due to Walras' law.

3.3.1 Solving the Model

A full solution of the model is provided in the appendix. We now focus only on some of the key conditions. We guess and later verify that, around the steady state, patient agents are lenders while impatient agents are borrowers. Optimal saving and capital investment decisions of lenders are characterized by:

$$\frac{u_1'(c_{1t})}{\zeta(R_t - 1) + 1} = \beta_1 \frac{u_1'(c_{1t+1})(1 + r_{t+1})}{\zeta(R_{t+1} - 1) + 1} \tag{7}$$

$$u_1'(c_{1t}) q_t \frac{\eta(R_t - 1) + 1}{\zeta(R_t - 1) + 1} = \beta_1 u_1'(c_{1t+1}) \frac{(R_{t+1} - 1)\eta q_{t+1} + e^{A_{t+1}} f'(k_{1,t}) + q_{t+1}}{\zeta(R_{t+1} - 1) + 1} \tag{8}$$

Equation (7) is lenders' Euler equation. The only difference with a traditional one is the use of an adjusted marginal utility, $\frac{u_1'(c_{1t})}{\zeta(R_t-1)+1}$, rather than just the marginal utility. The adjustment takes into account that a fraction ζ of consumption requires cash to be accumulated in advance and its opportunity costs is the interest rate. A standard Euler equation is recovered when $\zeta = 0$.

Equation (8) characterizes lenders' optimal capital choice. The left hand side is the marginal cost of buying one additional unit of capital at time t in terms of consumption good. The term $\frac{\eta(R_t-1)+1}{\zeta(R_t-1)+1}$ is the relative cost imposed by the CIA constraint. When $\zeta = \eta$ the relative cost is just one. If $\eta > \zeta$ and $R > 1$ then capital investments are relatively more costly than consumption goods. The right hand side is the marginal benefit of buying one additional unit of capital. It includes the marginal product of capital, the resale value of the capital and saving from having extra cash in the amount $(R_{t+1} - 1)\eta q_{t+1}$ without having to accumulate it in advance.

Consider next the borrowers' problem. Let $\Omega_{2,t}$ be the Lagrangian multiplier for borrowers' CIA constraints. Optimal saving and capital investment decisions are characterized by

$$u_2'(c_{2t}) = \zeta \Omega_{2,t} + (1 - \zeta)\beta_2 \Omega_{2,t+1} \frac{1}{1 + \pi_{t+1}} \tag{9}$$

$$\begin{aligned}
\Omega_{2,t}\eta q_t &= \beta_2\Omega_{2,t+1}\left(\frac{\gamma_t q_{t+1}}{R_{t+1}} + \eta q_{t+1} - \frac{1}{1+\pi_{t+1}}(1-\eta)q_t\right) \\
&\quad + \beta_2^2\Omega_{2,t+2}\frac{e^{A_{t+1}}f'(k_{2,t}) + (1-\eta-\gamma_t)q_{t+1}}{1+\pi_{t+2}}.
\end{aligned} \tag{10}$$

We define μ_t as the increase rate of money supply. Then

$$M_t^s = (1+\mu_t)M_{t-1}^s$$

and

$$t_t = \frac{\mu_t}{1+n}M_{t-1}$$

Government can choose either $\{t_t\}$ or $\{\mu_t\}$ as the instrument of monetary policy. We focus on μ_t later in this paper when we study monetary policy. We also have

$$nm_{1,t} + m_{2,t} = \frac{1+\mu_t}{1+\pi_t}\left[nm_{1,t-1} + m_{2,t-1}\right] \tag{11}$$

For purposes of comparison, it is convenient to characterize the equilibrium allocation in absence of collateral constraints. The following proposition states that the allocation of capital is efficient.

Proposition 2 $f'(k_{1,t}) = f'(k_{2,t}) = f'(\frac{\overline{K}}{1+n})$ for $t > 0$ when there is no collateral constraints.

Proof. See appendix. ∎

Without collateral constraints, the difference in discount factors leads only to differences in the allocation of consumption but not of capital. In particular, consumption of impatient individuals eventually drift towards zero while patient individuals consumes all the output. This requires borrowers to accumulate enough debt. As we show next, when collateral constraints are binding the allocation of capital is not efficient and consumption for borrowers and lenders is interior.

3.3.2 Steady State

We no characterize the steady state of the economy. Notation without subscript refers to steady state values. In the presence of collateral constraints, the following sufficient condition guarantees a unique steady state.

Assumption 2

$$(1 - \beta_2 + \pi) \eta + \beta_2 (1 - \beta_1 \gamma) > 0.$$

This assumption requires is relatively weak as it is satisfied by any positive inflation. It will also be satisfied by the optimal inflation, as shown in the next section.

Proposition 3 *Let Assumption 2 hold. Then, there exists a unique steady state. The steady state is characterized by $f'(k_2) > f'(k_1)$. In particular,*

$$\frac{f'(k_2)}{f'(k_1)} = \frac{\beta_1^2 [\eta(1 + \pi)(1 - \beta_2) + \beta_2(1 - \eta)(1 - \beta_2) - \beta_2(\beta_1 - \beta_2)\gamma]}{\beta_2^2(1 - \beta_1)[\eta(1 + \pi) + (1 - \eta)\beta_1]} > 1 \quad (12)$$

$$q = \frac{\beta_1^2}{(1 - \beta_1)[\eta(1 + \pi) + (1 - \eta)\beta_1]} f_1'(k_1). \quad (13)$$

$$c_2 = \frac{1}{1 + \zeta\pi} \left[f(k_2) - (1 - \beta_1)\gamma q k_2 + \frac{\pi}{1 + n}\zeta \left(nf(k_1) + f(k_2) \right) \right] \quad (14)$$

and

$$c_1 = \frac{nf(k_1) + f(k_2) - c_2}{n} \quad (15)$$

Proof. We first characterize the steady state. From (7) and (11):

$$R = \frac{1 + \pi}{\beta_1}, \text{ and} \quad (16)$$

$$\pi = \mu. \quad (17)$$

By using (8) and (16), we have (13). (10) gives

$$\frac{\beta_2 - \beta_2^2 \frac{1}{1+\pi} R}{R} \gamma q + \beta_2 \eta q + \beta_2^2 \frac{1}{1 + \pi} f'(k_2) + \beta_2^2 (1 - \eta)q \frac{1}{1 + \pi} = \eta q + \beta_2 (1 - \eta)q \frac{1}{1 + \pi}. \quad (18)$$

By (3), (11), (5) and government's monetary supply, we can get

$$(1+n)\frac{t}{p} = \frac{\pi}{1+\pi}\zeta[nf(k_1) + f(k_2)] \tag{19}$$

The budget constraint (2), CIA constraint (3) and collateral constraint (4) of agent 2 together generate

$$(1+\zeta\pi)c_2 + (1-\beta_1)\gamma q k_2 = f(k_2) + \frac{t}{p}(1+\pi) \tag{20}$$

We have

$$\overline{K} = nk_1 + k_2 \tag{21}$$

by (5).

$$nc_1 + c_2 = nf(k_1) + f(k_2) \tag{22}$$

is a result from market clearing of consumption. Cash in advance constraints and patient households' budget constraints generate

$$\zeta c_1 = m_1 \frac{1}{1+\pi} + \frac{t}{p} \tag{23}$$

$$\zeta c_2 = m_2 \frac{1}{1+\pi} + \frac{t}{p} \tag{24}$$

$$c_1 + m_1 + \frac{R}{1+\pi}b_1 = e^{A_t}f(k_1) + b_1 + m_1\frac{1}{1+\pi} + \frac{t}{p} \tag{25}$$

(6) means

$$nb_1 + b_2 = 0 \tag{26}$$

The equations above along with (13) characterize the steady state. From (13), (16), (17), (18), it follows that:

$$\frac{\beta_1^2[-\eta(1+\pi)(1-\beta_2) - \beta_2(1-\eta)(1-\beta_2) + \beta_2(\beta_1-\beta_2)\gamma]}{(1-\beta_1)[\eta(1+\pi) + (1-\eta)\beta_1]}f'(k_1) + \beta_2^2 f'(k_2) = 0.$$

Rearranging one obtains (12). We next show that show $\frac{f'(k_2)}{f'(k_1)} > 1$. According to (12), $f'(k_2) = f'(k_1)$ if $\beta_2 = \beta_1$. Since $\beta_2 < \beta_1$, it is sufficient to show that $\partial \frac{f'(k_2)}{f'(k_1)}/\partial\beta_2 < 0$ in equation (12). Notice that

$$\partial \frac{f'(k_2)}{f'(k_1)}/\partial\beta_2 = C \frac{-(1-\eta)\beta_2^2 - 2\beta_2\eta(1+\pi) + \beta_2^2\eta(1+\pi) + \beta_2^2\beta_1\gamma}{\beta_2^4}$$

where $C = \frac{\beta_1^2}{(1-\beta_1)[\eta(1+\pi)+(1-\eta)\beta_1]} > 0$. Assumption guarantees that the term $-(1-\eta)\beta_2^2 - 2\beta_2\eta(1+\pi) + \beta_2^2\eta(1+\pi) + \beta_2^2\beta_1\gamma$ is negative so that $\partial \frac{f'(k_2)}{f'(k_1)}/\partial\beta_2 < 0$. Existence and uniqueness of the steady state follows from the fact that $\frac{f'(k_2)}{f'\left((\overline{K}-k_2)/n\right)}$ spans the real line and it is strictly decreasing in k_2.

From (19) and (20), we have

$$(1 + \zeta\pi)c_2 + (1 - \beta_1)\gamma q k_2 = f(k_2) + \frac{\pi}{1+n}\zeta[nf(k_1) + f(k_2)]$$

Thus

$$c_2 = \frac{1}{1+\zeta\pi}\left[f(k_2) - (1-\beta_1)\gamma q k_2 + \frac{\pi}{1+n}\zeta\left(nf(k_1) + f(k_2)\right)\right]$$

which is (14). After that, we can easily get (15) by using (22) $\blacksquare$

Proposition 3 shows that there is always a productivity gap between borrowers and lenders when collateral constraints are binding. In particular, borrowers have higher marginal productivity. Figure 1 illustrates the steady. In absence of collateral constraints, $f'(k_1) = f'(k_2)$ and borrowers have capital k_2^*. When collateral constraints are binding, borrowers have k_2 and there is a gap between $f'(k_1)$ and $f'(k_2)$.

The effects of monetary policy on steady state allocations is fundamentally affected by the presence of the collateral constraints. In absence of collateral constraints, money will be superneutral in long term: inflation will have no effect on the distribution of capital or consumption, even if cash is needed for both consumption and investment. As we already discussed in Proposition 2, absent collateral constraints, equilibrium allocations satisfy $f'(k_{1,t}) = f'(k_{2,t}) = f'(\frac{\overline{K}}{1+n})$. In the appendix we also discuss that in equilibrium
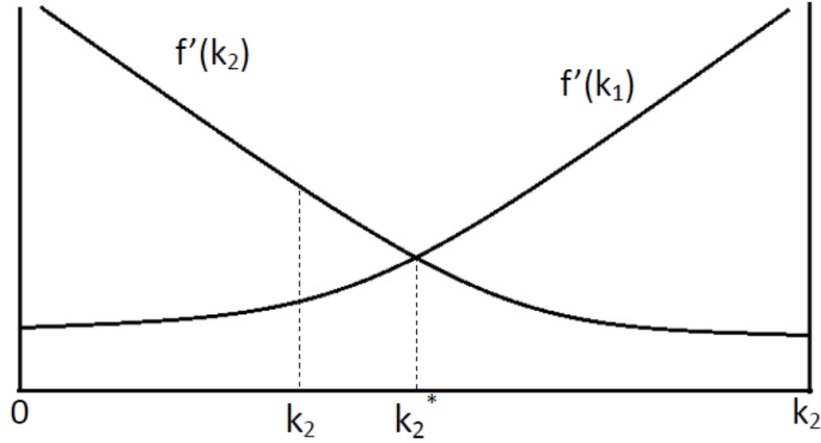
Figure 1: Productivity gap

borrowers' consumption drifts toward zero while lenders' consume all production. Thus, monetary policy does not affect the asymptotic equilibrium.

Consider now the equilibrium with binding collateral constraints. According to equation (12) in Proposition 2, k_2 is a negative function of the inflation as long as cash is needed for capital investments ($\eta > 0$). The intuition is as follows: the inflation tax makes capital accumulation more costly for both borrowers and lenders, but in the margin, it affects more borrowers than lenders. The reason is that, as shown in equation (13), the price of capital falls when the inflation is higher tightening credit constraints. The price of capital described by (13) is the present discounted value of the marginal product of capital. A higher inflation, lowers the price of capital due to the added cost of holding cash for its purchase. When $\eta = 0$, inflation does not affect the distribution of capital but it still affects the distribution of consumption, as long as cash is required for its purchase.

The degree of financial development of the economy is captured by γ. A larger γ describes a more developed financial system. As described by equation (12), a higher γ reduces the steady state productivity gap and improves efficiency of the economy. In addition to the direct effect of relaxing borrowing constraints, for a given value of the collateral, the improved allocation of capital also raises the price of capital, as shown by equation (13), inducing an even further relaxation of the credit limits. The direct and indirect effects could be quantitatively significant. Figure 2 show the results above.
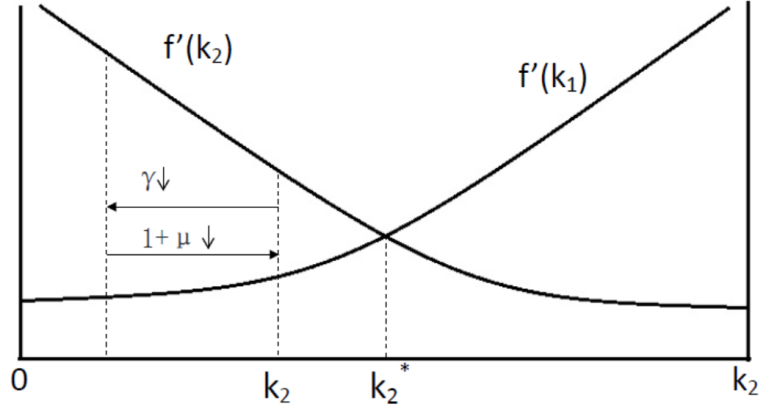
Figure 2: Change of financial conditions and monetary policy

3.3.3 Impulse Response Functions

We now study short term dynamics of the model around the steady state. We study impulse response functions to three types of unexpected shocks: a TFP shock (A-shock), a financial shock ($\gamma-$shock) and a monetary shock ($\mu-$shock). For this purpose, we perform numerical simulations[2].

Assume A_t and γ_t follow the processes:

$$A_t = \rho A_{t-1} + \epsilon_t, \text{ and}$$

$$\gamma_t = \delta \gamma_{t-1} + (1 - \delta)\gamma + \varepsilon_t,$$

where γ is the steady state of γ_t, $\epsilon_t \tilde{} N\left(0, \sigma_\epsilon^2\right)$, $\varepsilon_t \tilde{} N\left(0, \sigma_\varepsilon^2\right)$ and $Cov(\epsilon_t, \varepsilon_t) = 0$. Assume the utility function is $\ln(.)$ and the production function is standard neoclassical production function $e^{A_t} f_i(k_{i,t-1}) = e^{A_t} k_{i,t-1}^{0.7}$. We use the following parameters for our simulations $\beta_1 = 0.99$, $\beta_2 = 0.98$, $\eta = 0.6$, $\zeta = 0.95$, $\gamma = 0.995$, $n = 1$, $\overline{K} = 1$, $\rho = 0.65$, $\delta = 0.65$. Although the exact parameter values are important, and affect, the quantitative effects, the qualitative effects are overall robust to the exact parameters.

Consider first a TFP shock. Figure 3 shows the impulse responses of a TFP drop of 1% at time 0. From the figure, it is clear that the TFP shock in our model does not provides any surprise. One percent TFP shock causes a one percent decrease of production and

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[2]We use Dynare 4.5.1 to simulate our model.

consumption. Other real terms effect are quite small and can be ignored. There is no much amplification nor extra persistence beyond what is generated exogenously by the shock. These results are consistent with the findings of Cordoba and Ripoll (2004b).
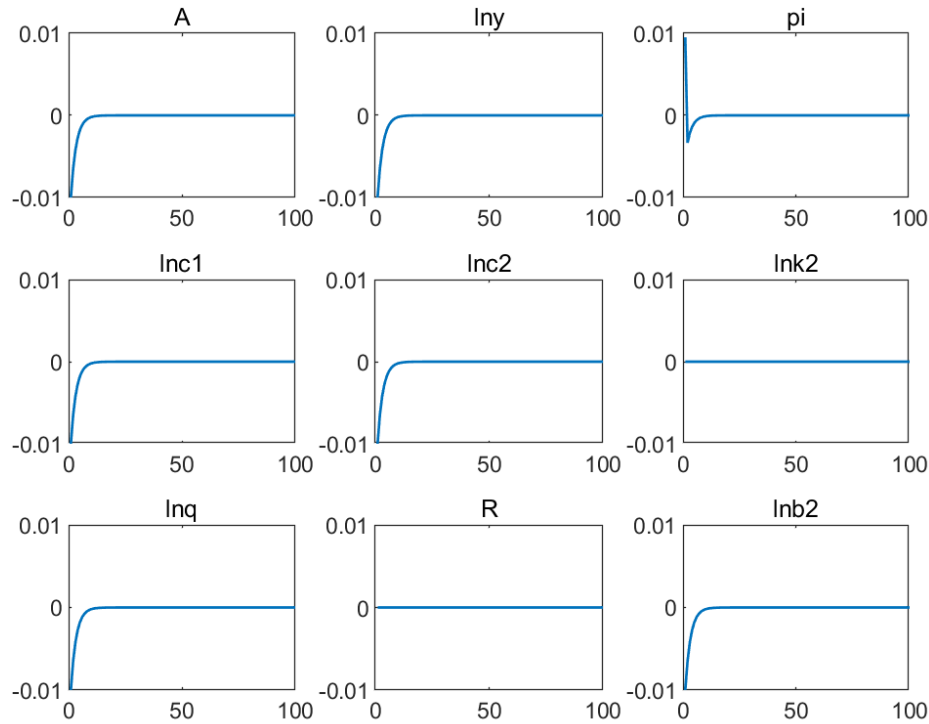

Figure 3: TFP shock

Consider next a financial shock. In particular, consider a shock at time zero that decrease γ_0 by 0.01, from the benchmark of 0.995 to 0.985. Figure 4 shows the corresponding impulse responses functions. We find that even a small shock to the financial sector, which quiet down in about 10 periods, can generate very persistent fluctuations. The recession of total output is not particularly large but much more persistent than the financial shock. There are also significant redistribution effects. Although consuming a little more in the very beginning, lenders in this economy soon bear a relative big loss. On the other hand, borrowers in this economy benefit from the financial shock, after an initial drop in consumption.

This result is a little counterintuitive. How can borrower benefit when they face tighter collateral constraints? When there is a financial shock and γ decreases, borrowers

cannot borrow as much as before with the same amount of capital. Then they have to reduce consumption for a bit because they cannot borrow as much as they used to. Interest rates also fall reflecting the drop in the demand for debt. Borrowers sell land seeking to smooth consumption. Land price begins to fall and so is production because borrowers have higher marginal productivity. Lenders' wealth decreases for both the land price and interest revenue decreases so they have to reduce their consumption.
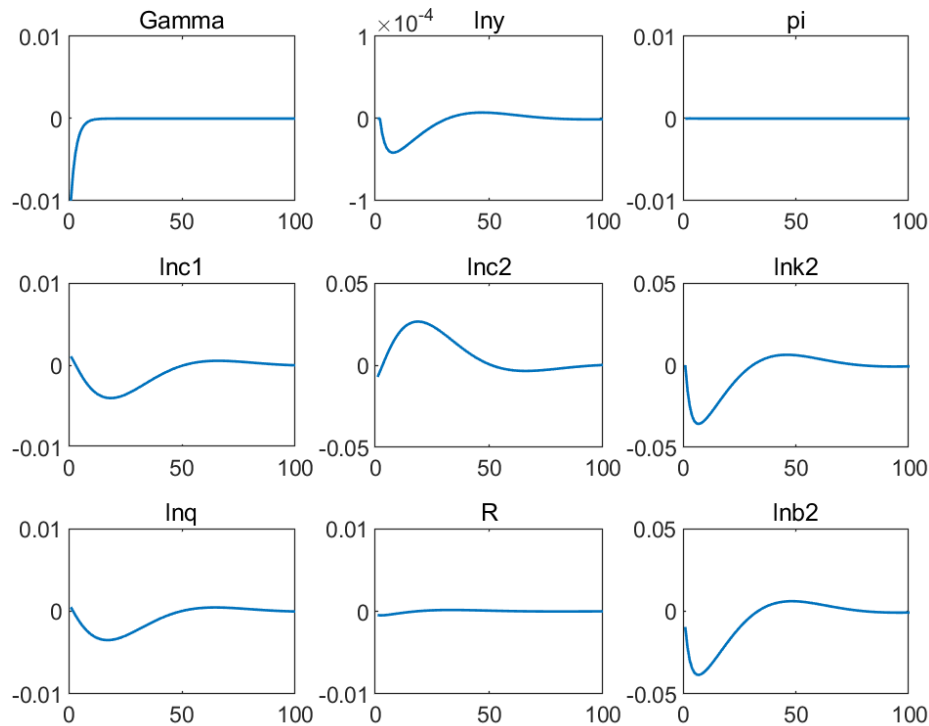
Figure 4: Financial shock

We then study the role of monetary policy. We increase the money increase rate $1 + \mu$ by 0.01.

From Figure 5, the money shock also generates fluctuations. An expansionary monetary policy lead to higher inflation and higher nominal interest rate. Lenders will reduce consumption while borrower increase consumption. Borrowers also reduce capital and debts and the asset price decrease. Since borrowers have more productivity, total output decreases.

Figure 5: Monetary shock

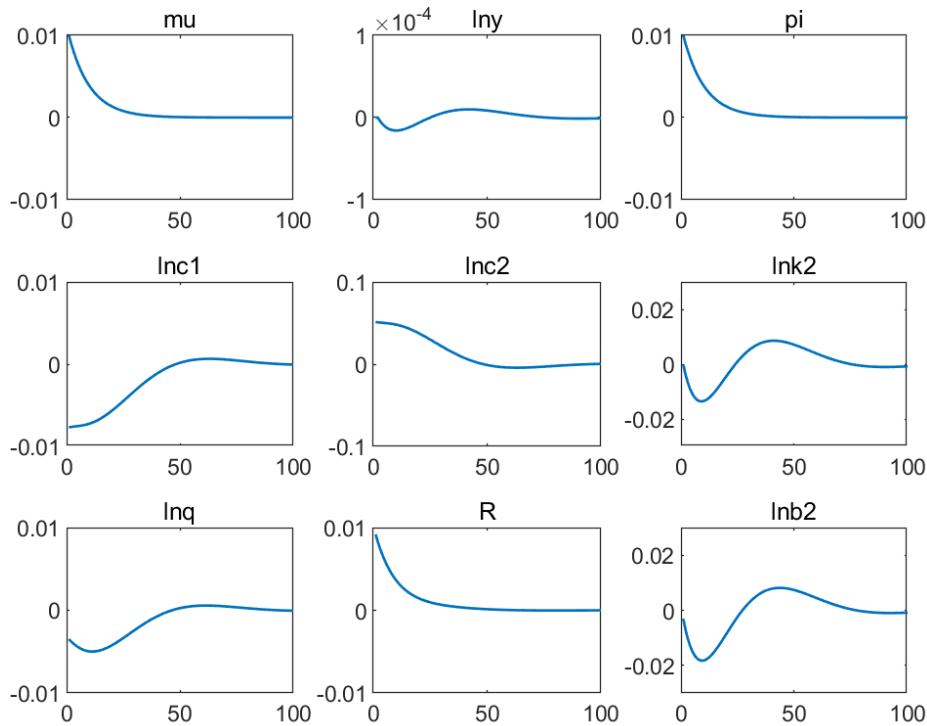Why does expansionary monetary policy generates this result? There are two channels monetary policy work through. The first one is it changes the cost of keeping cash. Just as we have discussed in section about steady state. Because there are cash in advance constraints, households have to keep some cash for future consumption and buying capital. Inflation reduces the purchasing power which means there is a cost of keeping cash. The higher inflation the higher cost keeping cash would be. Thus both lenders and borrowers tend to reduce the demand of capital when inflation rate increases. However, capital can only be used in producing by lenders while borrowers can also use capital to secure their debts. Since the demand of capital reduce the price of capital and borrowers cannot use the same amount of capital to borrow the same amount of debts. This lead to borrowers' demand of capital decreases more than the lenders' so borrowers sell capital to lenders which again decreases the debts secured and that's why we see there is a decrease in debt. Since borrowers have higher productivity than lenders, the reallocation of capital generates recession. And since borrowers do not need cash for capital any longer and can

even get some cash by selling capital they will increase their consumption. At the same time, buying capital reduce lenders' cash for consumption. What is worse, the decreasing price of capital and decreasing debt revenue also make lenders reduce consumption.

The second channel is redistribution of money. At steady state, lenders has much more cash than borrowers. In our simulation, lenders have about 5.7 times cash as borrowers have. When there is an unanticipated expansionary monetary shock by helicopter drop, both the lenders and borrowers have more cash than they plan to have in nominal term. However, the proportion of cash between borrowers and lenders increases because when there is an one percent increase of money supply the borrowers' cash increases more than one percent while lenders' increases less than one percent. Since an one percent shock generates one percent inflation lenders have less cash in real term and borrowers have more cash in real term. Thus an expansionary monetary policy dilute lenders' cash. Although this channel does exist, we find this redistribution effect is quite small. Things do not change much even we revise the helicopter drop monetary policy to a situation that helicopter drop base on the current holding of cash which means those who have 10% of money get 10% new money. (please see appendix) For this reason we mainly use first channel to discuss optimal monetary policy in next section.

3.4 Optimal Monetary Policy

We have found that the money is not superneutral in the long term and can have big effect on the economy in the short term. Thus it is natural to come to the question. What is the optimal monetary policy for the government? To answer this question, we use Ramsey policy to study optimal monetary policy. The setting of Ramsey policy is described as below.

In this section, we assume there is a benevolent government. The government does not consume. The only aim of the government is to maximize social welfare by using monetary policy and it cannot affect the economy except by using monetary policy. Thus, what the government faces is a Ramsey problem.

We assume the social welfare function is

$$E_0\alpha \sum_{t=0}^{\infty} \beta_1^t u_1(c_{1t}) + (1-\alpha) \sum_{t=0}^{\infty} \beta_2^t u_2(c_{i2t}) \tag{27}$$

where $0 < \alpha < 1$ is the weight of two types of agents' utility functions

Definition 4 *A Ramsey problem is to choose the competitive equilibrium defined in Definition 1 to maximize (27) by choosing optimal monetary policy $1 + \mu_t$.*

We can solve this Ramsey problem by using Lagrangian method, please see the appendix. Then we will use the result to study what's the optimal monetary policy in the long run by studying the steady state and in the short run by using simulations.

3.4.1 Steady State

Proposition 5 *Optimal monetary policy makes the Lagrangian multiplier for borrowers' collateral constraint $\nu = 0$ at steady state and the optimal money increase rate and inflation rate are*

$$1 + \mu = 1 + \pi = \frac{\beta_2(\gamma + \eta - 1)}{\eta} = \beta_2 \left(1 - \frac{1-\gamma}{\eta}\right) \tag{28}$$

if $\eta > 1 - \gamma$.

Proof. See Appendix. We can also easily check that optimal monetary policy satisfy Assumption 2. ∎

Our steady state optimal monetary policy is a long term optimal monetary policy. The most famous long term optimal monetary before is Friedman rule. Friedman (1969) maintains optimal monetary policy should make $R = 0$ because the social cost of keeping

cash is zero while the individual cost of keeping cash is R. The best optimal monetary policy should make the social cost and individual cost of keeping cash the same. Although other authors criticize Friedman rule such as Phelps (1973). They tend to discuss the optimal inflation rate with government spending. In our paper, however, even there is no government expenditure, we still show that Friedman rule is not the optimal monetary policy because the revenue of money is not zero for it can be used to reallocate capital. If cash is not needed to buy capital, monetary policy would loss its benefit and then Friedman rule holds. At the same time, notice that when $\gamma = 1$ and $\beta_2 \to \beta_1$, our optimal monetary policy also converge to Friedman rule because collateral constraints almost not bind in that situation so it comes to a situation close to Friedman rule. This two situation indicates Friedman rule works only in a special situation.

The Largragian multiplier $\nu = 0$ means that at the steady state the optimal monetary policy should make the shadow price of collateral be zero which means it cannot benefit the whole society any more if borrowers can borrow more. At this point, reducing more financial incompleteness is not the best choice for the society. However, this does not mean collateral constraints do not bind any more. By plugging optimal monetary policy into Largragian multiplier in appendix we find shadow price of collateral constraints for borrowers at steady state is $\varrho_2 = \beta_2 \Omega_2(\frac{1}{1+\mu} - \frac{\beta_2}{\beta_1^2})$. Substitute optimal monetary policy into it

$$\varrho_2 = \beta_2 \Omega_2\left(\frac{1}{1+\mu} - \frac{\beta_2}{\beta_1^2}\right) = \Omega_2 \frac{\eta\beta_1^2 - \beta_2^2(\gamma + \eta - 1)}{(\gamma + \eta - 1)\beta_1^2} > \Omega_2 \frac{\beta_2^2(1 - \gamma)}{(\gamma + \eta - 1)\beta_1^2} \geq 0$$

so shadow price for borrowers are still greater than zero. Borrowers are still facing binding collateral constraints though it is not so tight as before. We have showed that when $\gamma = 1$ and $\beta_2 \to \beta_1$, optimal monetary policy becomes Friedman rule. At that time, $\varrho_2 = \beta_2 \Omega_2(\frac{1}{1+\mu} - \frac{\beta_2}{\beta_1^2}) = 0$. It is because in that situation, borrowers and lenders converges to representative model. There are not borrowers and lenders any more so collateral constraints do not have effect.

We compare this model with model without collateral constraints. We have already discussed a model without collateral constraints when we discuss steady state in competi-

tive equilibrium. We conclude that money is superneutral at the asymptotic steady state. Thus there is no optimal monetary policy in long term without collateral constraints.

Why does not the government just choose monetary policy to make efficient production $f'(k_{1,t}) = f'(k_{2,t})$? If an government tries to do so, it leads to too strong reallocation of wealth. Although the total production does increase, the change of wealth leads to a decrease of social welfare. Inflation smaller than optimal monetary policy leads to a welfare loss.

For an economy with greater γ or an economy has more developed financial system, the government should choose more expansionary monetary policy to stimulate borrowers to accumulating capital. For an economy with smaller γ or an economy has less developed financial system, the government should choose more contractionary monetary policy.

From (28) and the steady state results (13), (16), (17), (18), (19), (20), (21) we have derived from competitive equilibrium section, we have

$$\frac{f'(k_2)}{f'(k_1)} = \frac{\beta_1^2 \beta_2 \gamma}{\beta_2^2[\beta_2 \gamma + (1-\eta)(\beta_1 - \beta_2)]} \tag{29}$$

$$q = \frac{\beta_1^2}{(1-\beta_1)[\beta_2 \gamma + (1-\eta)(\beta_1 - \beta_2)]} f'(k_1) \tag{30}$$

When γ increase both k_2 and q decrease. That's to say when using optimal monetary policy, an economy has greater γ has cheaper capital price and borrowers have less capital. The marginal productivity gap is greater with greater γ. These results are on the contrary to the original results with different γ in competitive equilibrium. Why is that?

For an economy with smaller γ, borrowers have to use more capital to secure their debts and the capital price is low as we have shown in section 3. Benefit of keeping capital for borrowers are less so the borrowers would not like to have much capital. It makes the collateral constraints bind more and the economy is inefficient. The government choose to use more contractionary monetary policy because this can offset the inefficient by increasing the capital price and capital borrowers kept as we have seen in section 3. Since γ is less, only with capital valued more it can make the economy runs efficient enough.

For the reason above, it seems that the government would like to choose an overshoot monetary policy.

3.4.2 Optimal Monetary Policy in Fluctuations

Now we turn to study how monetary policy can be used to deal with economic fluctuations. We have known from Section 3.2 that both technology shock and financial shock can generate fluctuations. In this section, we use the same parameters to do simulations to see how monetary policy can be used to eliminate fluctuations. Before report our result, we give a brief review about results of financial shock and monetary policy in competitive equilibrium. When there is a financial shock (i.e. γ_t decreases), it generates a long lasting fluctuations. There is both recession of output and redistribution effect between lenders and borrowers. Monetary policy mainly works through changing cost of keeping cash which can affect the demand of capital and price of capital. These together changes the benefit of keeping capital especially for borrowers which leads to a reallocation of total capital.

We find that monetary policy cannot be used to eliminated TFP shock. The reason is monetary policy in our model works through reallocation of capital while TFP shocks only have lump-sum effect. For the financial shock part, however, the monetary policy is very effective. A short periods contractionary monetary policy can totally eliminate the fluctuations of most real terms. Figure 6 is the impulse responses against γ shock with and without optimal monetary policy. Blue lines are impulse responses when there is a financial shock while the government does not response to the shock and still uses the optimal monetary policy at steady state. Red lines are the results when the government uses optimal monetary policy. We can see the optimal monetary policy is a short term contractionary monetary policy. This contractionary monetary policy reduces inflation which means the cost of keeping cash decreases (more strictly speaking, it is benefit of keeping cash increases because it would be deflation around optimal policy). This means the cost of keeping cash to buy capital decreases and the demand of capital increases. As we have discussed in last section, price of capital increases and the benefit of keeping cash
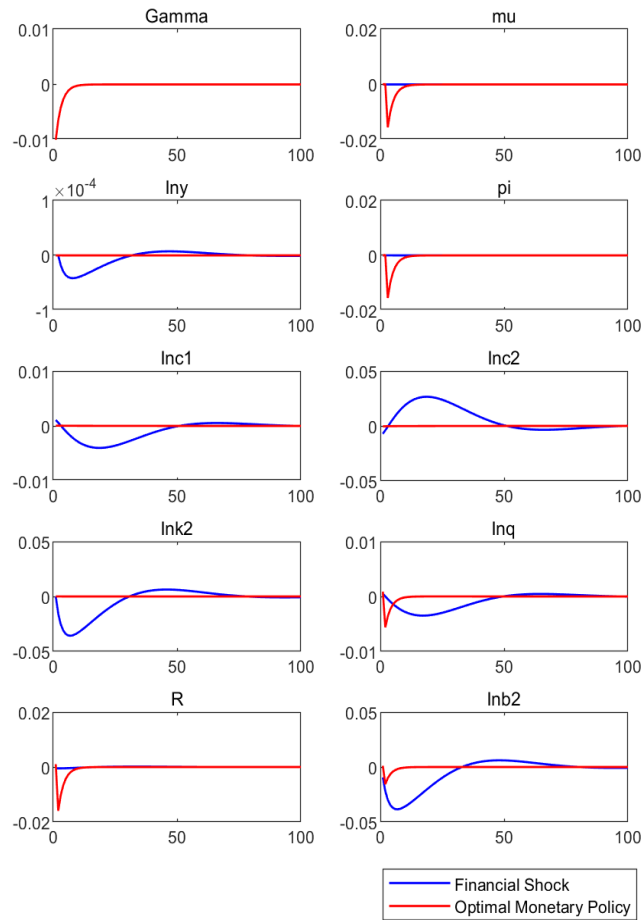
Figure 6: Effects of financial shock and optimal monetary policy

for the borrowers increases because a unit of capital can secure more debts. It offsets the trends of decreasing capital held by borrowers and recession is eliminated. Both lenders and borrowers will consume the same amount of goods as before with the optimal monetary policy.

3.5 Conclusion

In this paper, we study how monetary policy works when financial market is incomplete. We find that financial frictions such as collateral constraints can misallocate social resource both in long run and short run. At the same time, monetary policy have a strong redistribution effect through reducing cost of purchasing capital. Monetary policy plays a significant role in helping the economy and increasing social welfare if there is inefficiency caused by distortion of resource. Failing to take this redistribution channel into

consideration monetary policy may not achieve the goal expected by policy maker. Thus, different countries should set different aim of inflation rate in the long run according to their financial system. There is no certain best inflation rate for every country. When government want to reduce business cycles by business cycles, they must also understand what property of the business cycle is since monetary is much more effective to deal with financial shock than TFP shock.

Though our model is very simple, it does provide a way to understand monetary policy which can be used in every heterogeneous agents literature with money. The way we find the optimal monetary policy can be valid in literatures about heterogeneity and money. The channel of monetary policy works through we discuss worth consideration in other heterogeneous models.

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CHAPTER 4. BLOCKCHAIN INNOVATION IN GLOBAL PAYMENTS: NETWORK EFFECTS, CREATIVE DESTRUCTION AND BUBBLES

4.1 Introduction

The global payments industry is large and plays an important role in world economy. According to Niederkorn etal. (2016), global payments industry generated $1.8 trillion in revenue in 2015 and it is estimated to generate $2.2 trillion in revenue in 2020. Cross-border payments comprised about 40 percent of total global payments transactional revenues (i.e., transaction-related fees and float income) while they accounted for less than 20 percent of total payments volumes. Cross-border payments generated $300 billion in global revenue. However, global payments, especially cross border payments, are costly and inefficiency. According to the Remittance Prices Worldwide Report of the World Bank (2017), the global average cost for sending USD 500 was 4.56 percent in Q4 2017. While the internet has facilitated and reduced the costs of international communications enormously, the speed of sending remittance cross-border remains very slow and may often take two days or longer. For example, it takes less than 14 hours to fly from New York to Beijing which means that it is faster to take cash by plane from New York to Beijing than to use traditional remittance methods.

Recent technological advances have shown potential to disrupt, or significantly alter the global payment system that has been characterized by sluggish technological progress. Financial institutions and fintech companies started to use blockchain technology. Blockchain uses cryptography and information technology to record data in open and distributed ledgers that could significantly increase the security and efficiency of information transfer and transactions. The basic concept of blockchain was first described in early 1990s (Haber and Stornetta, 1991) but there is no big progress and real product until Nakamoto wrote his paper (Nakamoto, 2008) and developed Bitcoin in 2009. Bitcoin relative success have stimulated a myriad of copycats but also new products. Although many blockchain products may not have any real potential, others may do. If successful, blockchain technologies could reshape the global payments system.

A fintech company, Ripple, was the first to utilize blockchain technology with the purpose of reducing cost and increase efficiency of the payment industry. Founded in 2012, Ripple developed a protocol capable of making cross-border and cross-currency transaction in less than four seconds. The transaction costs can be extreme small. According to a report by Ripple itself (2016), Ripple can cut bank's global settlement costs up to sixty percent. There have been an increasing number of financial institutions working with Ripple. Among them are many well known brands such as American Express, Western Union, Moneygram, UBS, Santander, CIBC, BBVA, UniCredit and Standard Chartered based on Ripple's official website. Besides Ripple, there are several competitors seeking to use blockchain technology in the global payments system. R3, for example, is a company leading a consortium of more than 80 big banks and has its own distributed ledger/blockchain platform Corda. The incumbent giant for global payments is SWIFT, an institution founded in 1973 and currently linking more than 11000 financial institutions worldwide. SWIFT is also embracing blockchain technology by developing SWIFT GPI. It reports that nearly 50% of payments can be credited to end beneficiaries in less than 30 minutes (SWIFT 2018). These examples show that industry insiders consider blockchain technologies as serious alternatives with the potential to reshape the global payments industry.

To gain some idea about the degree of ongoing technological progress, consider the following. Currently Bitcoin may take more than sixteen hours to finish one transaction and the cost per transaction can be as high as $55 due to the costly "mining" process. Today, Ripple only need less than four seconds with extreme small transaction cost, usually far less than one cent according to XRP charts which is official live and historical data website about Ripple network. Since blockchain technologies are still in an early stage of development and adoption, there is still significant room for technological progress and adoption. As time goes on, it is possible that more competitors providing new solutions for global payments will arise. What is the future of the global payment industry given the disruptive nature of technological progress occuring? Will the traditional technology be displaced, or could the incumbent retain its market share by adopting new blockchain

technologies such as SWIFT GPI? Or other leaders such as Ripple and R3 will grow to attract more financial institutions into their network? Or a new leader will emerge with a new technology?

Technological innovations, particularly disruptive ones, often correlate with speculative bubbles (Shiller 2000). Many economists believe there are bubbles in blockchain industry. For example, the price of a Bitcoin was just $0.06 in July 19th, 2010, $6.18 on January 2nd, 2012, and more than $19000 on December 16th, 2017. At the same time, Bitcoin is not widely used. Except some black market use case, there are very few transactions using Bitcoin. Many economists believe the intrinsic value of Bitcoin is much less than it is now so the price of Bitcoin is bubble. Bitcoin is not the exception. We have already seen thousands of blockchain technologies and more than 1500 cryptocurrencies in the market. Most of cryptocurrencies do not have a real world use now and they do not have clear roadmap or business plan to be used in the near future. However, the all time high market cap for cryptocurrencies up to now is 813 billion while the maximum of trade volume in exchange in 24 hours is 44 billion according to coinmarketcap.com. These suggest that many cryptocurrencies have few chance to succeed in business traded in high prices, much higher than their intrinsic value. This is what economists define as bubbles. For example, Dogecoin, a copycat of Bitcoin, has attracted a large number of speculators though the developers themselves did not regarded it as a serious project. Dogecoin has not been updated since 2015 which makes its technology lags far behind, with little if any business use and not much potential. However, its market cap increased from less than 1.5 million USD in 2015 to 2 billion USD on January 7th, 2018. Though it is difficult to conclude that cryptocurrencies are bubbles, there is still a sense that some cryptocurrencies, such as Dogecoin, are. We investigate what effects bubbles have on technological progress in global payments. Do they stimulate technological progress or do they generate a net loss to society?

This paper builds a theoretical model to shed light on some of these questions. To reflect a key characteristic of global payments and blockchain technology, this paper uses the economic concept of a network effect. A network effect is the positive effect

that an additional user of a product has on the value of that product to others. The value of the product depends on how many users there are. Both global payments and blockchain technology have a clear network effect. When a financial institution uses a product for international payments, it has to cooperate with other financial institutions directly or indirectly. Thus the more financial institutions there are, the more convenient the payment is. Blockchain technologies also show network effects. They need more participants to record transactions on ledgers. If there are very few participants, the cost for participants to falsify data is low. In an extreme case, if an recorder has more than 50% share of recording power, every claim it made will be accepted as truth in the blockchain network. Even though no one has that big power, It is relatively easy for the recorder to claim transactions which do not exist if it is a big enough recorder thus there is security problem. An increasing number of recorder can significantly reduce such risk.

There are two sectors in our model: an R&D sector and financial institutions. Every period inventors in the R&D sector have a chance to discover a new better technology and optimally choose whether or not to develop it into a real network product that can be used by financial institutions. Financial institutions choose whether or not to join a network within their information set. Information about the set of available products is imperfect and diffuses slowly. The more financial institutions there are in one network, the more efficient it is. The slow diffusion of information plays an important role. If information diffuses instantly the solution is trivial. All financial institutions join the leading network and abandon older networks at the same time. In this case, network effects need not to be considered. When there is imperfect diffusion of information, there is heterogeneity among financial institutions and network effects play key role. Thus we find diffusion of information is important when considering network effect and technological progress.

In our basic model technological progress is exogenous but its adoption is endogenous. We characterize the dynamics of technological progress as a three stage process. In the first stage, the state of the "blockchain" technology is not advanced enough so that new discoveries are not developed into real network projects and financial institutions use the traditional technology. A second stage of "creative destruction" starts when the level

of knowledge is sufficiently large. Inventors in the R&D sector find that some of their products can attract a critical mass of financial institutions, enough to compensate for the costs of developing a new network. This Schumpeterian process of "creative destruction" continues as some new technologies develop into real projects and some old technologies lose their market share. In the end, one or more blockchain networks are both advanced enough and big enough so their market share never fall. This is the third stage and we call it formation of oligarch.

Then we extend our model to allow for endogenous technological progress requiring a research cost. We find that unless the initial level of knowledge is large enough as to allow initial innovations to break even, blockchain technologies will not be developed. Direct government R&D investments could bring the frontier of knowledge to the required critical level. Another alternative that could help ignite innovation are bubbles. Bubbles make technological progress possible since even projects with no chance to be used by financial institutions could still get enough return for technological progress to occur. This is in fact consistent with the recent history of blockchain technology. Although basic concepts have existed for a while, technological progress was very slow and there was no major progress until Bitcoin came along. Many regard Bitcoin price to be fundamentally a speculative bubble. But this bubble has ignited an innovation frenzy and several years after that there are thousands of new products of blockchain technology. According to our model, bubbles can help technological progress but they may also encourage excessive development of networks that have no chance of business success, which decreases social welfare. Thus, bubbles may increase or decrease social welfare depending on the dominant effect.

There is a large literature on network effects. Primary concerns of this literature is the notion critical mass and the dynamics and stableness of critical mass (Jackson and Yariv 2007). Many papers focus on price theory and competition in two-side or multi-sided platforms (Indirect network effect) such as Rochet and Tirole (2003), Armstrong (2006) and Weyl(2010). Some papers discuss technology adoption and innovation. One topic is adoption of technology such as Kazt and Shapiro(1986). There are very few papers about

R&D or introducing new technologies with network effects. Kristiansen (1998) studies a two firms two buyers three stages game. Shy (1996) uses an OLG model to study the demand of a durable product which has network effects and technological progress. However, every generation is trapped in the sense that it has to use a certain good and cannot switch as the generation gets older. Moreover, there are at most two products existing at the same time and the share of them totally depends on the population of each generation. Thus, technology progress occurs only as new generations arrive. This mechanism cannot really account for the fast and disruptive nature of progress observed in many industries with network effects. Similar to the papers just mentioned, our model has the adaption of new technology and the concept of critical mass. At the same time, our paper contributes to the understanding of the dynamic process of technological progress with network effects.

Our paper also relates to the blockchain literature. Though this topic is popular in public media and engineering fields, there are still not many studies in economics. Numerous papers discuss Bitcoin's technology, its potential use, risk and governance issues (Bohme, Christin, Edelman, Moore 2015, Davidson, De Filippi, Potts 2016, Catalini and Gans 2016) but without solid theoretical basis. Among the few economic papers with solid foundations are: Cong and He (2018) who study how smart contract using blockchain technology can affect competition; and Barrdear and Kumhof (2016) who study the effect if central government chooses to issue digital currencies as a second legal tender. However, to the extent of our knowledge, there is no paper focusing on global payments and there is no paper discussing network effect of blockchain technology in a theoretical way.

Finally, our paper relates to the literatures on bubbles. There is a large literature of the relationship between investment and bubbles. For example, Farhi and Tirole (2012) study under which conditions bubbles are more likely to emerge and they also find that bubbles crowd investment in (out) when liquidity is abundant (scarce). Hirano and Yanagawa (2017) study the relationship between asset bubbles and endogenous growth and they also find that bubbles have crowd in and crowd out effects on investment and growth. Different with their models, our model is a partial equilibrium model for one

certain industry. However, in our extension, we still find bubbles have crowd in effect to increase the investment and technological progress in blockchain technology use in global payment industry.

In section 2, we describe our basic model in detail and the results are given in section 3. We extend the basic model in section 4 and section 5 is the conclusion.

4.2 The Basic Model

4.2.1 Timeline

As we mention in introduction,there are two kinds of agents in our basic model: inventors (R&D sector) and financial institutions. Inventors find and develop new technologies which can help reduce payments cost while financial institutions pay license fee to use inventors' blockchain network to do global payments. Before we describe inventors' and financial institutions' behavior in detail, we first describe the timeline of one period to help understand the model.
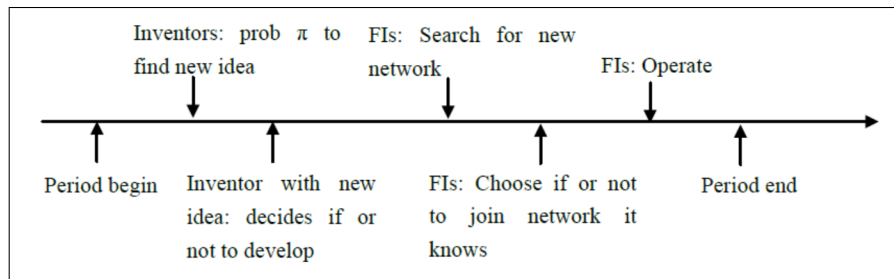

Figure 1: Timeline

As the Figure 1 shows, inventors act first and then financial institutions act. From the beginning of a period, nature decides if inventors find an new idea of blockchain technology and which inventor finds the idea. If there is new idea, the inventor who finds the idea decides whether to develops it into a real product. After that come the financial institutions. They first search for new network (products) they do not know before. Financial institutions have heterogeneous knowledge. Then they decide if they want to pay license fee to join some networks they know but have not joined before. If they join it they do not have to use it immediately but have the right to use it from then on. In the last step financial institutions operate to use networks which can help them to

get most profit this period to do payments. After that this period ends and a new period begins.

4.2.2 R&D Sector

We assume there are many potential inventors who can invent new blockchain technologies to reduce transaction cost of global payments. Here the cost contains both money cost and time cost. In the beginning ($t = 0$), there is no blockchain technology for global payments and the only technology being used by all financial institutions is traditional technology. At the beginning of the first period, $t = 1$, one of the inventors has the idea of first blockchain technology. After that, every period there is probability π that another person finds an idea better than the best idea before. In the basic model, we assume only doing scientific research to find a new idea is costless. Later we will extend our model to including cost of scientific research. We assume the jth technology has productivity $A^{1-\frac{1}{j}}$

If an inventor has an idea j at time $t(j)$, he has to decide whether he wants to develop it into a real project which can be used and paid by financial institutions. If he decides not to develop it, he publishes his idea and other inventors can use it as reference to make progress and have better idea. If he decides to develop it, he has to pay R&D cost which is c. The inventor also has to decide the license fee τ_j he wants financial institutions to pay. If a financial institution chooses to pay the license fee, it can use the network anytime it wants to use after that. $m_{j,t} = \int 1_{i,j,t} di$ is the measure of financial institutions pay the license fee and join the blockchain j at time t where $1_{i,j,t}$ is an indicator function. The indicator function is 1 when financial institution i decides to join j blockchain at time t. Otherwise $1_{i,j,t}$ is 0. Total revenue of the project is $\tau_j \sum_{t=t(j)}^{\infty} \beta^{t-t(j)} m_{j,t}$. Where β is the discount rate. If the inventor decides to develop a project, his expected profit is $E_{t(j)}\Pi_j = E_{t(j)}\tau_j \sum_{t=t(j)}^{\infty} \beta^{t-t(j)} m_{j,t} - c$.

Since there are many inventors we assume it is a monopolistic competitive market. Other inventors can imitate its project and provides similar product. To prevent competitor from entering the market, the inventor must set license fee to make

$$E_{t(j)}\Pi_j = E_{t(j)}\tau_j \sum_{t=t(j)}^{\infty} \beta^{t-t(j)} m_{j,t} - c = 0 \tag{1}$$

The inventor chooses to develop it only when he can set τ_j to make expected profit be zero and chooses not to develop it when it is impossible to find such license fee. We use indicator Υ_j to denote if there is τ_j to satisfy (1). $\Upsilon_j = 1$ if there exists such τ_j, otherwise $\Upsilon_j = 0$. From definition,

$$\Upsilon_j E_{t(j)}\Pi_j = \Upsilon_j \left[E_{t(j)}\tau_j \sum_{t=t(j)}^{\infty} \beta^{t-t(j)} m_{j,t} - c \right] = 0 \tag{2}$$

4.2.3 Financial Institutions

There are a continuum of financial institutions $i \in [0,1]$. An institution does three steps every period. The financial institution first tries to search if there are potential new blockchain networks. They have the probability $\delta > 0$ to find the true value and usage of a certain blockchain network every period and before that they cannot join it. δ is the parameter of diffusion of information. As we discuss in introduction, δ is the key to generate network effect. If $\delta = 1$, all financial institutions have the same information and they will use and give up using a certain blockchain network at the same time so the network effect does not matter very much. If $\delta < 1$, network effect affects the choice. The smaller the δ is, the greater the information heterogeneity is. And the increasing of heterogeneity increases the role of network effect. We use $\Omega_{i,t}$ to denote the network financial institution i knows at the first step of period t. We assume which network a financial institution has known and joined is public information. We use Θ_t to denote the public information.

After that, financial institution decides whether or not to join this blockchain network. It costs τ_j to join the blockchain and it can use this blockchain network to send remittance

any time it wants after joining it. If it refuses to join the network, it still has chance to join every period in the second step. We use $J_{i,t}$ to denote the set of feasible technology for financial institution i at period t. If i has joined blockchain network j then $j \in J_{i,t}$. Otherwise $j \notin J_{i,t}$. We use $\Gamma(J_{i,t}, J_{i,t-1})$ to denote the total license fee financial institution i has to pay when their feasible set of technologies changes from $J_{i,t-1}$ to $J_{i,t}$.

The third step is to operate their normal business. For simplicity, we assume the revenue for an institution is always the same as R. A financial institution chooses traditional technology or any blockchain networks they have already joined to do payment. For simplicity, we assume the cost of traditional technology is fixed as ε. The cost of technology j is $g\left(A^{1-\frac{1}{j}}, S_{j,t}\right)$ where $S_{j,t}$ is the measure of institutions which have already joined blockchain j at time t. $S_{j,t} = \sum_{l=t(j)}^{t} m_{j,l}$. Function $g(.)$ satisfy properties $g_1 < 0, g_2 < 0$. $g_{11} \geq 0, g_{22} \geq 0, \lim g_1(A,.) \to 0, \lim g_2(.,1) \to 0$. It is very intuitive. It is trivial to assume a certain technology which is more advanced has lower cost. The second property comes from network effects. If there are very few institutions join a blockchain network, it is very inconvenient to use it to send remittance so it is inefficient and cost financial institutions more. The more institutions there are, the more efficiency there are. Although better technology and more participants do reduce cost, their marginal effect is decreasing. When technology has already reduce the transaction time to less than one second, new technology which is able to reduce more time cannot reduce much more cost. If there have already been 90% financial institutions using the network, they do not benefit much from one more financial institution. Thus the cost function is $C_{i,t} = \min\left\{\min_{j \in J_{i,t}}\left\{g\left(A^{1-\frac{1}{j}}, S_{j,t}\right)\right\}, \varepsilon\right\}$ and financial institution problem in step 3 is

$$\max\{R - C_{i,t}\} \Leftrightarrow \min\left\{\min_{j \in J_{i,t}}\left\{g\left(A^{1-\frac{1}{j}}, S_{j,t}\right)\right\}, \varepsilon\right\}$$

If we combine all the three steps, we can write the following Bellman equation to indicate financial institution i's choice in period t.

$$V\left(J_{i,t-1}, \Omega_{i,t}, \Theta_t\right) = \max_{J_{i,t}}\left\{\begin{array}{c} R - \min\left\{\min_{j \in J_{i,t}}\left\{g\left(A^{1-\frac{1}{j}}, S_{j,t}\right)\right\}, \varepsilon\right\} \\ -\Gamma(J_{i,t}, J_{i,t-1}) + \beta E_t\left[V\left(J_{i,t}, \Omega_{i,t+1}, \Theta_{t+1}\right) | \Omega_{i,t}, \Theta_t\right] \end{array}\right\} \quad (3)$$

4.2.4 Equilibrium

Given the description of R&D sector and financial institutions, we can define the equilibrium.

Definition 1 *An equilibrium of the model is given information set $\{\Omega_{i,t}, \Theta_t\}$, prices $\{\tau_j\}$, indicators $\{\Upsilon_j\}$, accessible network set $\{J_{i,t}\}$, measure of joining network at certain time $\{m_{j,t}\}$ and measure of joining network in total $\{S_{j,t}\}$ solve inventors' problem (2) and financial institutions' problem (3).*

4.3 Result of the Basic Model

Before we come to the main propositions of this paper. We first set a lower bound of τ_j. An inventor never develops an idea unless he can at least get zero profit. To achieve this goal, he has to set the price of τ_j to at least satisfy $\frac{\delta \tau_j}{1-\beta(1-\delta)} \geq \frac{\delta \tau_{\min}}{1-\beta(1-\delta)} = c$ which means he does not get negative profit in the best situation in which all financial institutions are in at the time they find this technology. Thus

$$\tau_{\min} = c \frac{1 - \beta(1-\delta)}{\delta}$$

We first show that there exists some technology which is not advanced enough so inventors do not invest to develop them into products. Thus blockchain technology cannot replace traditional technology in the early stage.

Proposition 2 *if an idea j satisfies*

$$j < \underline{j} = \frac{1}{1 - \ln_A g_{S=1}^{-1} \left(\varepsilon - (1-\beta)\tau_{\min}\right)}$$

it will never be developed into a real project. Here $g_{S=1}^{-1}(.)$ is the inverse function of g given $S = 1$.

Proof. If the inventor develops idea j they have to set price $\tau_j \geq \tau_{\min}$. If financial institutions choose to join the blockchain j, they pay τ_j. The benefit of using the technology, however, is less than $\sum_{t=0}^{\infty} \beta^t \left\{ \varepsilon - g\left(A^{1-\frac{1}{j}}, 1 \right) \right\}$. If $j < \underline{j} = \frac{1}{1 - \ln_A g_{S=1}^{-1} (\varepsilon - (1-\beta)\tau_{\min})}$,

$\sum_{t=0}^{\infty} \beta^t \left\{ \varepsilon - g\left(A^{1-\frac{1}{j}}, 1\right) \right\} < \tau_{\min}$. Even all institutions join the network and use it forever the revenue is still less than the possible lowest license fee $\tau_{\min}$. Thus, no one wants to join the blockchain and the inventor cannot get revenue to satisfy (1). ∎

This proposition is a sufficient condition, though not necessary condition, to show that there is some blockchain technology are not useful for global payments. Any blockchain technology like this will cause net wealth loss for the whole economy if they are developed. In an economy without bubble, no one wants to develop them into projects.

We then show the existence of ideas developed into projects and adopted by some financial institutions.

Proposition 3 *There is at least one idea which have been developed by inventors and adapted by some financial institutions before or when idea $j^{\#}$ is invented while idea $j^{\#}$ is the smallest j satisfy*

$$\sum_{t=t^*}^{\infty} \beta^t \left(1-\pi\right)^{t-t^*} \left[\varepsilon - g\left(A^{1-\frac{1}{j}}, 1 - \left(1-\delta\right)^{t+1-t^*}\right)\right] > \tau^*$$

where

$$t^* = \arg\min_t \left\{ g\left(A^{1-\frac{1}{j}}, 1 - \left(1-\delta\right)^{t+1-t(j)}\right) \leq \varepsilon \right\}$$

and

$$\tau^* \left[\begin{array}{c} \beta^{t^*-t(j)} \left(1-\pi\right)^{t^*-t(j)} \left(1 - \left(1-\delta\right)^{t^*-t(j)+1}\right) \\ + \sum_{t=t^*+1}^{\infty} \beta^{t-t(j)} \left(1-\pi\right)^{t-t(j)} \delta \left(1-\delta\right)^{t-t(j)} \end{array} \right] = c$$

Proof. Assume there is no idea has been developed and adapted before. When idea $j^{\#}$ appears, if the inventor decides to develop it and set the license fee as τ^*, the expected benefit for financial institutions which have found its true value at the moment when it saves more cost than traditional technology and join the blockchain is

$$E_{t(j)} R_f > \sum_{t=t^*}^{\infty} \beta^t \left(1-\pi\right)^{t-t^*} \left[\varepsilon - g\left(A^{1-\frac{1}{j}}, 1 - \left(1-\delta\right)^{t+1-t^*}\right)\right] > \tau^*$$

so they certainly join the blockchain. Here the right hand side of first inequility is only the part of expected cost saving to calculate the situation when there is no new idea

exsits. Thus total expected benefit for financial institutions must be greater than this one. That's the logic behind the first inequility. And the expected revenue of inventor is

$$E_{t(j)}R_i > \tau^* \left[\begin{array}{l} \beta^{t^*-t(j)} (1-\pi)^{t^*-t(j)} \left(1 - (1-\delta)^{t^*-t(j)+1} \right) \\ + \sum_{t=t^*+1}^{\infty} \beta^{t-t(j)} (1-\pi)^{t-t(j)} \delta (1-\delta)^{t-t(j)} \end{array} \right] = c$$

The inventor of $j^\#$ then has positive profit to develop the idea so in fact he set the license fee $\tau_{j^\#} < \tau^*$ to satisfy the zero profit condition. Then the profits for financial institutions are even higher so they join the network. Thus, there is at least an idea has been developed and adopted when idea $j^\#$ is invented. ∎

We have already found that there are blockchain technologies can be successful in business. The next question is as technological progress new technologies may replace old ones so is it possible that some technologies can reach a point that no potential competitor has chance to attract their consumers? Next two propositions provide both sufficient condition for such blockchain networks and proof of the existence of such networks.

Proposition 4 *If an blockchain network j has share S_j satisfy*

$$g\left(A^{1-\frac{1}{j}}, S_j\right) < g(A, 1) + (1-\beta)\tau_{\min}$$

financial institutions in blockchain network j will never try any other blockchain networks.

Proof. We consider an extreme case all others not joining blockchain j before first discover the best technology and join the best technology immediately after they find it. Also, we assume first group of firms in blockchain j join the best blockchain immediately, the inventors can get highest return in this case. The license fee τ_A still has to satisfy $\tau_A \geq \tau_{\min} = c\frac{1-\beta(1-\delta)}{\delta}$. If $g\left(A^{1-\frac{1}{j}}, S_j\right) < g(A, 1) + (1-\beta)\tau_{\min}$, we have $\sum_{t=0}^{\infty} \beta^t \left(g\left(A^{1-\frac{1}{j}}, S_j\right) - g(A, 1) \right) < \tau_{\min}$. Even this unrealistic low entry fee is still greater than the cost saved by the best technology, financial institutions in blockchian j never find it profitable to switch. Thus financial institutions in blockchain network j will never try any other blockchain networks. ∎
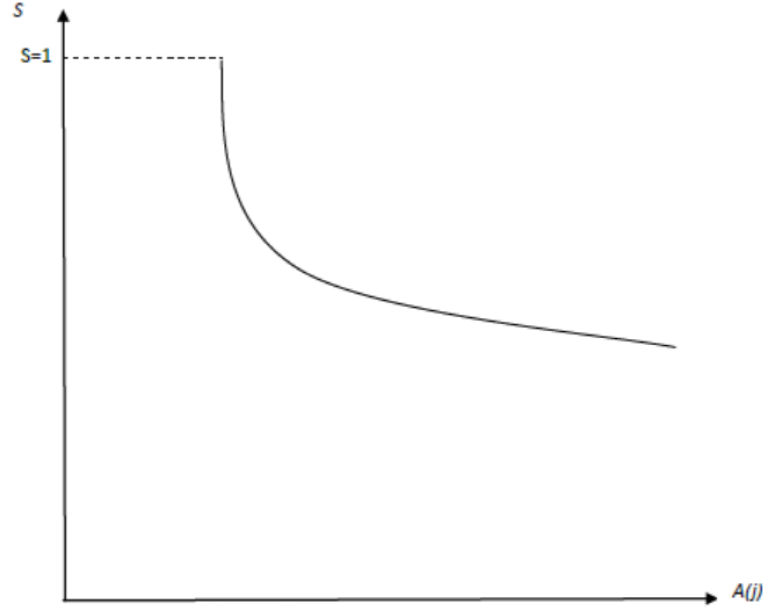
Figure 2: Region that product will keep its market share

Figure 2 describes this sufficient condition. Both market share and technology are important. Given the technology, the bigger the market share the easier it is to reach the sufficient condition. Given the market share, the more advanced the technology the easier it is to reach the sufficient condition. This sufficient condition shows that both network effect and technology itself can help a blockchain network survive in the long run.

Proposition 5 *There is at least one blockchain technology that can maintain its share and never collapses.*

Proof. For any infinite small number ϵ, we can always find a j^* such that for any share S, $g\left(A^{1-\frac{1}{j^*}}, S\right) - g\left(A, S\right) < (1 - \beta)\epsilon$. We assume there is no blockchain technology can maintain its share before j^*, otherwise we have already found what we want to show. If j^* is good enough to be developed into an project and financial institutions think it is profitable to join the network, it is never replaced by a better technology. Thus it is a blockchain network which never collapses. If it is not good enough to be developed compared with existing technology, since $g\left(A^{1-\frac{1}{j^*}}, S\right) - g\left(A, S\right) < \epsilon$, better technology is not good enough either. Which means the past technology will not be replaced by anything new so there must have already been one technology can survive forever. ∎

From propositions above, we understand how technological progress happens in global payments industry with blockchain technology. It can be divided into three stages. The first stage is early stage. Technology is not advanced enough in early stage so new discoveries are not developed into real projects. That is the intuition proposition 1 tells us. And this is very realistic and easy to understand. Traditional technology is more competitive when new technology is at the early stage.

However, it is not the end of the story. Technological progress happens and blockchain technology is getting more and more efficient so we come to the second stage which is "creative destruction" stage. Technologies ultimately reach a threshold when the level of knowledge is sufficiently large. Above the threshold technologies have the chances to be successful in business. But not all the technologies above the threshold can be developed into real projects. Whether a certain technology is developed into a project is determined by the existing networks and their market share. An existing project may operate for some time but replaced by others and even those financial institutions join it gradually abandon using it. We call this process "creative destruction" just as Schumpeter suggested. However, these technology shall not be viewed as bubbles even they may lose its market share soon because they have intrinsic value at least for some time and have potential to reduce global payments cost and increase social welfare. That is why financial institutions may want to pay to join it.

After this creative destruction process running for some time, there may be blockchain networks which are advanced and big enough so no one can attract financial institutions in this blockchain to pay license fee to switch to other blockchain. The whole global payments market are gradually dominated by one or several such networks depending on the realization of history. We call this third stage as formation of oligarch.

Figure 3 shows propositions and the technological process we discussed before. Propositions help us to characterize the whole process but they are only sufficient conditions or proof of existence rather than the thresholds itself. The process is too complicated so it is impossible to give the analytical threshold of each stage. However, these propositions are enough for us to understand the whole technological progress.
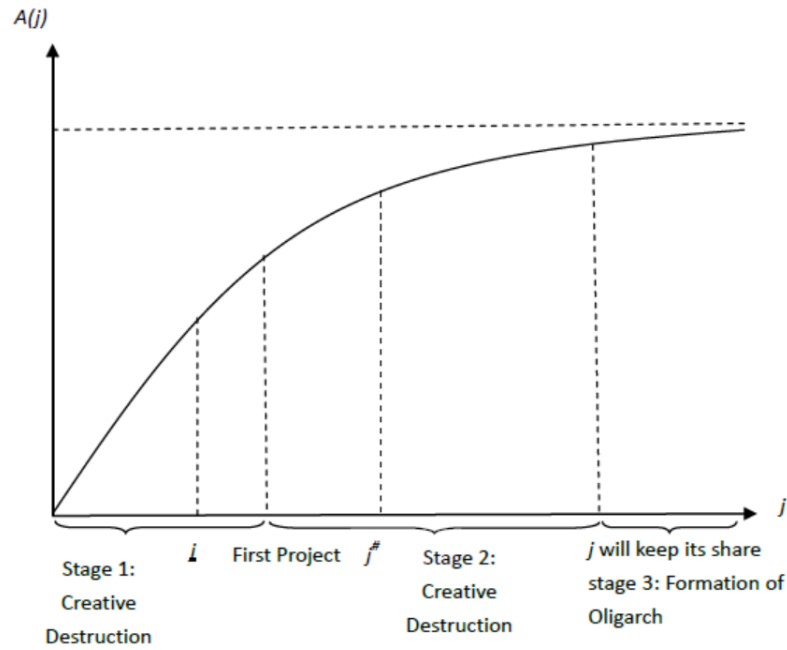
Figure 3: Evolution history of the industry

3.4 Extension: Research Cost, Government Funding and Bubbles

4.4.1 Research Cost

In the basic model, we assume that new ideas come into inventors' mind for free. Some people may think it is arbitrage to make such assumption. We know sometimes even pure academic research needs funding. In reality, it may take inventors several years to develop an preliminary idea which can be developed into a real business product. In this section we extend our model to see what happens if inventors have to pay some cost to even get an feasible idea. The technological progress in this section is endogenous rather than a total exogenous process.

We assume the inventor who has an new idea pay research cost ζ to get the preliminary result. Only with the preliminary result the idea can be developed into a real business project though the inventor may not choose to develop it. At the same time new research must stand on the preliminary result to find new idea. If the inventor choose not to do any research to get the preliminary result, other inventors know nothing about this idea and the economy behaves as if there is no such an idea. Technological progress still needs first find this idea.

Cost ζ in this extension may be not big in this extension but even a very small ζ has huge effect on the whole process. From proposition 2, when technology is not advanced enough no one wants to adapt the technology thus inventors get nothing but only pay the cost. If the initial idea is not profitable no one wants to pay the research cost ζ and blockchain technology will stay at the lowest level forever. There is no technological progress and blockchain technology never becomes useful. Also, when there has already some technologies existed, the innovation process will stop if a new idea is not profitable since the older ones have big share.

Though the intuition is very simple, it is just what happened before the invention of Bitcoin. The basic concept exists but technological progress is very slow, if there is some progress, that for almost twenty years there is no significant progress even in research. One of the most important reason for this situation is very few people want to invest in something that have almost zero return.

4.4.2 Government Funding

We know in many scientific research the private return is usually very small and the cost may be large. To encourage researchers to do research which can benefit the social welfare in the long term, governments devise some incentive system such as research funding. These funding can compensate the cost in research so researchers are willing to do research which the private return is less than the research cost. In our model, if a government funds an inventor ζ when an inventor publish his idea j then the process is just the same as the basic model and the total social welfare is ζj less than the basic model when idea j is found. Since we assume ζ is small, the total welfare loss is also small. In this ideal world, the problem is easy to solve with a small social cost.

However, we find it is not that easy to have a government working like that. For some new born technology, it is hard for governments to fully understand the value of the technology thus they may not choose to fund research in this field. As we have discussed, for nearly 20 years, few people understood the value of blockchain so it is impossible for governments to fund research of blockchain technology.

4.4.3 Bubbles

In reality, rapid technological progress happens right after the increasing price of Bitcoin. After Bitcoin is developed and attract some people's interest, the price of Bitcoin explodes. According to coindesk.com, in July 19, 2010, price of Bitcoin was just \$0.06. In less than one year and a half, price was \$6.18 on Jan 2, 2012. The price rose more than 100 times. After that, we see huge amount of new blockchain technology have been developed in several years, including all of the most important blockchain technology for global payments such as Ripple, R3 and SWIFT GPI. In this process, we have already seen thousands of blockchain technology and more than 1500 cryptocurrencies in the market. As we have discussed in introduction, clearly many cryptocurrencies are traded at prices which are much more than their intrinsic value. Those cryptocurrencies with prices much higher than their intrinsic value are bubbles. Most economists view bubbles as bad things which will harm financial systems and even whole economy badly. However, in this section, we use our model with research cost and bubbles to show that bubbles may help the process of R&D.

We assume in this model if the first blockchain technology is developed, no matter it is a successful business project or not, it attracts some attention and get extra return from third party besides license fee from financial institutions. Since the fundamental value of an technology is the cost financial institutions reduce and some of the welfare reallocates to inventors through license fee, the extra return is not related to fundamental value of a technology thus it can be viewed as bubbles. For simplicity, we assume every project has same bubble b_t at period t. We assume every period bubbles grow as the rate as η with probability ι. Here we have $\eta\beta > 1$. There is probability $1 - \iota$ that bubbles burst so the value of bubbles come to zero. This bursting bubble model is nothing new and it is first used by Blanchard (1979).

Now we can find that the expected profit of research to find an idea and develops it into a project becomes

$$E_{t(j)}\Pi = E_{t(j)}\tau_j \sum_{t=t(j)}^{\infty} \beta^{t-t(j)}m_{j,t} + B_j - c - \zeta$$

where $B_j = b_{t(j)} + \sum_{t=t(j)+1}^{\infty} (\beta\iota)^{t-t(j)} \eta^{t-t(j)-1} (\eta - 1) b_{t(j)}$ is inventor j's expected revenue from bubbles. if $B_j \geq c + \zeta$, inventor j always chooses to pay research cost and develops it into a real projects regardless of how many financial institutions choosing to join.

This extension of model does not affect the result of proposition 4 and proposition 5. It does change proposition 2 and proposition 3 because every new idea will be developed into real projects. However, there are many projects no financial institution will use.

Thus, bubbles have two effects. The first effect is positive and is a kind of crowd in effect. As we have seen, bubbles encourages R&D. This is just what happens in blockchain technology. Very few people pay attention to blockchain technology for nearly two decades but many inventors trying to develop new projects when there are bubbles in cryptocurriencies because they can get high return from bubbles. The technological progress begins until one day some projects which can change the global payments come into life and the social welfare increases.

However, bubbles also have negative effect. Many projects which are developed when there are bubbles are not profitable at all. If the project cannot attract enough financial institutions so $E_{t(j)}\tau_j \sum_{t=t(j)}^{\infty} \beta^{t-t(j)}m_{j,t} - c < 0$ for any reasonable τ_j, the optimal choice for the whole society is just finding the idea but not developing it into a project. However, with $B_j \geq c + \zeta$, inventor has incentive to develops it thus it is a welfare loss.

Thus the effects of bubbles are ambiguous. On the one hand it increases social welfare by encouraging technological progress, on the other hand it helps some projects which have no use and cause a deadweight loss. We have to decide which effect is dominant. According to data in our introduction, global payments industry is big and so is the cost. Since now we have already had product can reduce 60% of the cost we believe the

positive effect is the dominant effect thus bubbles in blockchain technology increase the social welfare.

4.5 Conclusion

In this paper, we study the evolution of blockchain technology in global payments industry theoretically. We find that technological progress and network effect both play significantly role. Generally speaking, the industry meets three stages, at the first stage technologies is not advanced enough and technologies cannot have business success. At the second stages, technologies are able to help reducing cost of global payments. Better technology can replace old ones although network effect give current leaders some advantages but it is not enough to compensate the technological weakness. At the final stage, some advanced technology with big network effect are unbeatable so one monopolist or several oligarchs dominate the industry.

We also find that technological progress can be slowed down for a long time even if research costs are small. Government funding can solve this problem but it maybe difficult for the government to understand the value of new technologies particularly at the very early stage. For example, governments did not choose to fund blockchain technologies when the technology was new but there is a growing interest by central banks to do so.

An alternative way to solve the problem are bubbles, which is consistent with the actual evolution of the blockchain industry. Bubbles attract speculator's attention and developers can get high return to compensate the R&D cost, sparking thus a wave of innovation. Bubbles have both positive effect to encourage technological progress but also have negative effects because they misallocate some resource to some technology which never have chance to succeed in business. Since the global payments industry is big, the positive effect is likely more important and blockchain technology could increase social welfare by reducing cost of global payments.

Although our paper focus on global payments and blockchain technology, the basic model can be used in many other fields with dynamic network effects such as the Internet. To the extent of our knowledge, our paper is the first to characterize the dynamic evolution

of an industry characterized by endogeneous technological progress and network effects. In this model, we find the diffusion of information plays an key role in discussing network effect. For simplicity we assume the speed of diffusion is a fixed parameter. Introducing a more complicated diffusion of information process, as it is done in a related network literature, will allow the model to account for other type of adoption strategies. Our R&D sector is also very simple but allow us to create a bridge between growth models and network effects. This paper also makes an contribution on the role of bubbles for innovation. Bubbles have played a role in at early stages in different industries, as it seems to be the case with blockchain technologies. For example, internet industry in the late 1990s and early 2000s had dotcom bubbles. Although many companies disappeared after the burst of bubbles, some now well known companies such as Google, were born during that period. Our mechanism provides a new channel to discuss bubbles in new born industries. Bubbles attract funding which can encourage technological progress.

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CHAPTER 5. GENERAL CONCLUSIONS

In my dissertation. I find that financial frictions hurt economy no matter for the whole economy or a certain industry. However, chapter 2 finds that monetary policy can be used to reduce financial frictions. Although traditional views on bubbles are bubbles are bad for economy, both chapter 3 and chapter 4 of my dissertation find that bubbles may be good when there are financial frictions especially in R&D sector.

In the future, I want to build some models which are more related with real data to study the effects of financial frictions. All the three chapters I in this dissertation are theoretical work. They provide interesting mechanism and intuition to explain how financial frictions affect on the economy and how different policy can be used to offset their negative effects. However, it is very important to do some empirical studies before these mechanism and intuition are used by policy maker. Thus, models provide theoretical background for empirical research are needed.

APPENDIX A. APPENDIX FOR CHAPTER 1

A.1 Proof of Proposition 6

Assume solution of R&D firm j's problem is $V_t\left(K_t^j\right) = a_t K_t^j + B_t$. We have

$$
\begin{aligned}
a_t K_t^j + B_t = \max_{K_{t+1}^j, K_{It+1}^j, I_t^j, B_t^j} & (1-\theta) r_t K_t^j + (1-\theta) q_t (1-\delta) K_t^j \\
& + (1-\theta) \rho_{t+1} B B_{t+1} + (1-\theta)(1-\pi)\left[-q_t K_{t+1}^j + \rho_{t+1} a_{t+1} K_{t+1}^j\right] \\
& + (1-\theta)\pi \left[\begin{array}{c} (q_t - 1)\left(r_t K_t^j + \rho_{t+1} a_{t+1} \xi (1-\delta) K_t^j + \rho_{t+1} B_{t+1}\right) \\ -q_t K_{It+1}^j + \rho_{t+1} a_{t+1} K_{It+1}^j \end{array}\right] \\
& + \theta a_t^\# K_t^j
\end{aligned}
$$

By taking first order derivative of K_{t+1}^j we have $q_t = \rho_{t+1} a_{t+1}$. By comparing the left hand side and right hand side we get

$$
a_t = (1-\theta) r_t + (1-\theta) q_t (1-\delta) + (1-\theta)\pi (q_t - 1)\left[r_t + \rho_{t+1} a_{t+1} \xi (1-\delta)\right] + \theta a_t^\#
$$

$$
B_t = (1-\theta)\left[1 + \pi (q_t - 1)\right] \rho_{t+1} B_{t+1}
$$

which are the proposition.

A.2 Derivation of Reallocation Effects Model

A.2.1 Households

The only difference is the budget constraints are now

$$
C_t + \int \left(V_t^j - D_t^j\right) \psi_{t+1}^j dj = \int V_t^j \psi_t^j dj + W_t \bar{L}
$$

A.2.2 Final Goods Producer

Since now the technology is

$$
Y_t = A \int_{n=1}^{N_t} (X_t^n)^\sigma (L_t^Y)^{1-\sigma} dn, 0 < \sigma < 1
$$

, profit maximization problem of the final goods producer is

$$\max A\int_{n=1}^{N_t} (X_t^n)^{\sigma} (L_t^Y)^{1-\sigma} dn - \int_{n=1}^{N_t} P_t^n X_t^n dn - W_t L_t^Y$$

subject to the production function. It is easy to solve profit maximization problem and we have the demand function for intermediate goods n

$$P_t^n = \sigma A (X_t^n)^{\sigma-1} (L_t^Y)^{1-\sigma}$$

$$W_t = (1-\sigma) A\int_{n=1}^{N_t} (X_t^n)^{\sigma} (L_t^Y)^{-\sigma} dn$$

A.2.3 Intermediate Goods Producers

Producer of Intermediate good n has profit

$$(P_t^n - 1) X_t^n = \sigma A (X_t^n)^{\sigma} (L_t^Y)^{1-\sigma} - X_t^n$$

Since we have already had intermediate goods n's demand function, we can find the price intermediate goods producer of goods n set.

$$P_t^n = \frac{1}{\sigma}$$

and the amount the producer produces

$$X_t^n = \sigma^{\frac{2}{1-\sigma}} A^{\frac{1}{1-\sigma}} L_t^Y \tag{1}$$

Then the profit of producing goods n every period is

$$\left(\frac{1-\sigma}{\sigma}\right) \sigma^{\frac{2}{1-\sigma}} A^{\frac{1}{1-\sigma}} L_t^Y$$

. Since we know that it is competitive monopolistic market, the discounted total profits from selling goods n must be equal to the cost of buying patent to produce goods n,

which means

$$\sum_{s=t}^{\infty} \rho\left(s,t\right)\left(\frac{1-\sigma}{\sigma}\right)\sigma^{\frac{2}{1-\sigma}}A^{\frac{1}{1-\sigma}}L_t^Y = \eta_n$$

Here $\rho\left(s,t\right) = \prod_{v=t+1}^{s}\left(\rho_{v+1}\right)$ if $s \neq t$, $\rho\left(s,t\right) = 1$ if $s = t$. Since only variables in η_n are time variables, patents created in the same period have the same price. This result gives us

$$\sum_{s=t}^{\infty} \rho\left(s,t\right)\left(\frac{1-\sigma}{\sigma}\right)\sigma^{\frac{2}{1-\sigma}}A^{\frac{1}{1-\sigma}}L_t^Y = \eta_n = \eta_t \tag{2}$$

A.2.4 R&D Sector

Same as baseline model.

A.2.5 Competitive Equilibrium

Definition 1 *A competitive equilibrium is defined as allocations*

$\left\{Y_t, K_t, C_t, I_t, N_t, E_t^j, T_t, L_t^j, L_t^Y, I_t^j, K_t^j, T_t^j, Y_t^i, \psi_t^j, X_t^n\right\}$ and prices

$\left\{w_t, P_n^t, R_t^j, q_t, \eta_t, r_t, V_t^j\right\}$ such that household maximize its utility and firms in all three sectors maximize their profits and market clearing conditions are satisfied which are stock market is clearing $\psi_t^j = 1$, labor market is clearing $\int_0^1 L_t^j dj + L_t^Y = \bar{L}$, debt market is clearing $\int_0^1 E_t^j dj = 0$, capital market is clearing $K_{t+1} = (1-\delta)K_t + I_t$, goods market are clearing $C_t + \int_{n=0}^{N_t}\int_0^1 X_t^n dn + I_t = Y_t$ and the amount of patent follows $N_{t+1} = N_t + T_t$.

A.2.6 Detrended Dynamic System

The detrended dynamic system now becomes

$$X_t = \sigma^{\frac{2}{1-\sigma}}A^{\frac{1}{1-\sigma}}L_t^Y$$

$$\sum_{s=t}^{\infty} \rho\left(s,t\right)\left(\frac{1-\sigma}{\sigma}\right)\sigma^{\frac{2}{1-\sigma}}A^{\frac{1}{1-\sigma}}L_t^Y = \eta_n = \eta_t$$

$$\rho_{t+1} = \beta\frac{c_t}{c_{t+1}\left(1+g_{t+1}^c\right)}$$

$$r_t k_t = Z\eta_t\left(k_t\right)^\alpha - w_t\left(\bar{L}-L_t^Y\right)$$
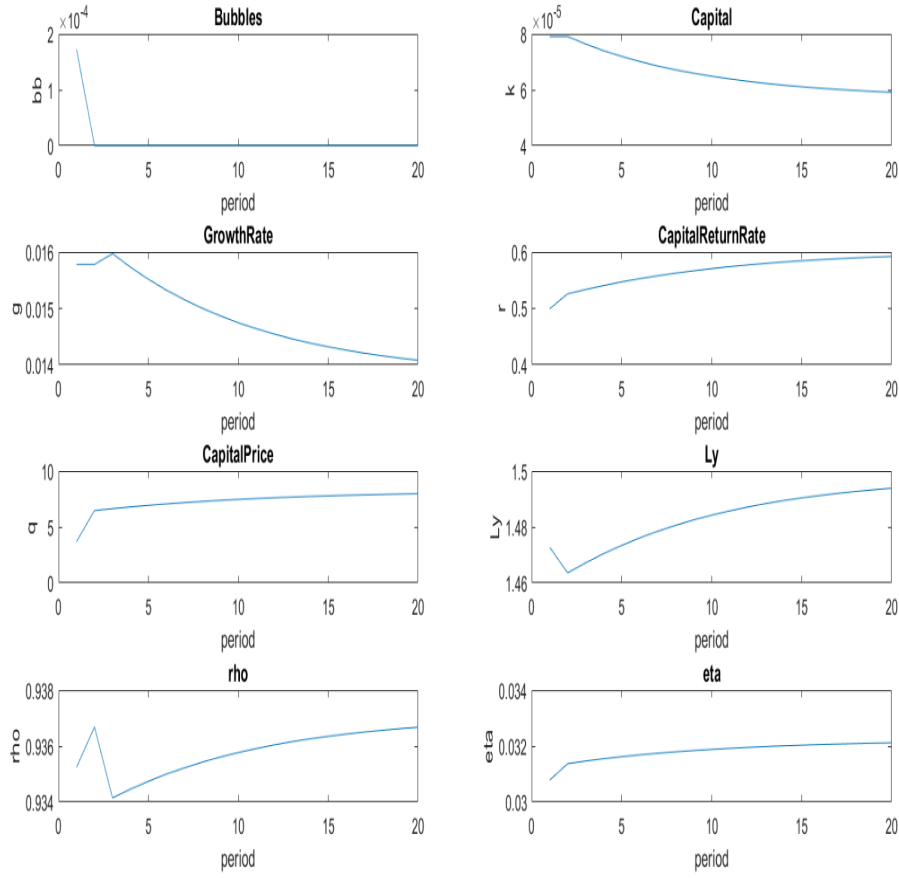
Figure 1: Burst of stochastic bubbles in reallocation effects model

$$a_t = r_t + q_t \left(1 - \delta\right) + \pi \left(q_t - 1\right) \left(r_t + \rho_{t+1} a_{t+1} \xi \left(1 - \delta\right)\right)$$

$$q_t = \rho_{t+1} a_{t+1}$$

$$b_t = \left[1 + \pi \left(q_t - 1\right)\right] \rho_{t+1} \left(1 + g_{t+1}\right) b_{t+1}$$

$$g_{t+1}^N = t_t$$

$$t_t = Z k_t^\alpha \left(\bar{L} - L_t^Y\right)^{1-\alpha}$$

$$(1 + g_{t+1}^N) k_{t+1} = (1 - \delta) k_t + \pi r_t k_t + \pi \rho_{t+1} \left[a_{t+1} \left(\xi \left(1 - \delta\right)\right) k_t + (1 + g_{t+1}^N) b_{t+1}\right]$$

$$r_t = \alpha \left(Z \eta_t\right)^{\frac{1}{\alpha}} \left(\frac{w_t \left(\bar{L} - L_t^Y\right)^\alpha}{1 - \alpha}\right)^{\frac{\alpha - 1}{\alpha}}$$

$$c_t + X_t + \pi r_t k_t + \pi \rho_{t+1} \left[a_{t+1} \left(\xi \left(1 - \delta\right)\right) k_t + (1 + g_{t+1}^N) b_{t+1}\right] = A X_t^\sigma$$

$$w_t = (1 - \sigma) A (X_t)^\sigma (L_t^Y)^{-\sigma}$$

A.2.7 Stochastic Bubbles Burst

We also study the case when bubbles burst stochastically. The setting is similar with stochastic bubbles in baseline model. We only report the simulation result here in figure 1. Just as the relationship between stochastic burst and unanticipated burst in baseline model, the pattern of stochastic burst and unanticipated burst are similar. The intuition is also similar with intuition we discuss before.

APPENDIX B. APPENDIX FOR CHAPTER 2

B.1 Derivation of F.O.Cs

We use Largrangian method to solve the model. For convenience, we have

$$m_{i,t} + \frac{R_t b_{i,t-1}}{(1+\pi_t)} + (1-\eta)q_t(k_{i,t} - k_{i,t-1}) + (1-\zeta)c_{i,t} = e^{A_t}f(k_{i,t-1}) + b_{i,t}$$

rather than budget itself.

$$
\begin{aligned}
L &= E_0 \sum_{t=0}^{\infty} \beta_i^t u_i(c_{it}) + \beta_i^t \Omega_{i,t}\big[\frac{m_{i,t-1}}{(1+\pi_t)} + \frac{t_t}{p_t} - \eta q_t(k_{i,t} - k_{i,t-1}) - \zeta c_{i,t}\big] + \\
&\quad \sum_{t=0}^{\infty} \beta_i^t \lambda_{i,t}\big[e^{A_t}f(k_{i,t-1}) + b_{i,t} - m_{i,t} - \frac{R_t b_{i,t-1}}{(1+\pi_t)} - (1-\eta)q_t(k_{i,t} - k_{i,t-1}) - (1-\zeta)c_{i,t}\big] \\
&\quad + \sum_{t=0}^{\infty} \beta_i^t \varrho_{i,t}\big[\gamma_t q_{t+1}(1+\pi_{t+1})k_{i,t} - R_{t+1}b_{i,t}\big]
\end{aligned}
$$

FOCs:

$$c_{it} : u_i'(c_{it}) - \zeta\Omega_{i,t} - (1-\zeta)\lambda_{i,t} = 0 \tag{3}$$

$$m_{i,t} : \beta_i \Omega_{i,t+1}\frac{1}{1+\pi_{t+1}} - \lambda_{i,t} = 0 \tag{4}$$

$$b_{i,t} : \lambda_{i,t} - \varrho_{i,t}R_{t+1} - \beta_i \lambda_{i,t+1}R_{t+1}\frac{1}{1+\pi_{t+1}} = 0 \tag{5}$$

$$k_{i,t} : -\Omega_{i,t}\eta q_t - \lambda_{i,t}(1-\eta)q_t + \varrho_{i,t}\gamma_t q_{t+1}(1+\pi_{t+1})+$$

$$\beta_i \Omega_{i,t+1}\eta q_{t+1} + \beta_i \lambda_{i,t+1}e^{A_t}f'(k_{i,t}) + \beta_i \lambda_{i,t+1}(1-\eta)q_{t+1} = 0 \tag{6}$$

For the lenders (type 1 agents), $\varrho_{1,t} = 0$. Using (3), (4), (5) to get (??)

$$\frac{u_1'(c_{1t})}{\zeta R_t + 1 - \zeta} = \beta_1 \frac{u_1'(c_{1t+1})(1+r_{t+1})}{\zeta R_{t+1} + 1 - \zeta}$$

(3), (4), (5), (6) to get

$$u_1'(c_{1t})q_t\frac{\eta R_t + 1 - \eta}{\zeta R_t + 1 - \zeta} = \frac{\beta_1 u_1'(c_{1t+1})[R_{t+1}\eta q_{t+1} + e^{A_{t+1}}f'(k_{1,t}) + (1-\eta)q_{t+1}]}{\zeta R_{t+1} + 1 - \zeta}$$

For the borrowers, substitute (4) into (3)

$$u_2'(c_{2t}) = \zeta\Omega_{2,t} + (1-\zeta)\beta_2\Omega_{2,t+1}\frac{1}{1+\pi_{t+1}}$$

from (5)

$$\varrho_{2,t} = \frac{\lambda_{2,t} - \beta_2\lambda_{2,t+1}R_{t+1}\frac{1}{1+\pi_{t+1}}}{R_{t+1}}$$

By using (4)

$$\varrho_{2,t} = \frac{\beta_2\Omega_{2,t+1}\frac{1}{1+\pi_{t+1}} - \beta_2^2\Omega_{2,t+2}R_{t+1}\frac{1}{1+\pi_{t+1}}\frac{1}{1+\pi_{t+2}}}{R_{t+1}} \tag{7}$$

substitute it into (6) we have

$$\begin{aligned}\Omega_{2,t}\eta q_t &= \beta_2\Omega_{2,t+1}[\frac{\gamma_t q_{t+1}}{R_{t+1}} + \eta q_{t+1} - \frac{1}{1+\pi_{t+1}}(1-\eta)q_t]\\ &+\beta_2^2\Omega_{2,t+2}\frac{e^{A_{t+1}}f_2'(k_{2,t}) + (1-\eta-\gamma_t)q_{t+1}}{1+\pi_{t+2}}\end{aligned}$$

B.2 Proof of Proposition 2

$$\sum_{t=0}^{\infty}\beta_i^t u_i(c_{it})$$

$$q_t(k_{i,t} - k_{i,t-1}) + c_{i,t} + m_{i,t} + \frac{R_t b_{i,t-1}}{1+\pi_t} = e^{A_t}f_i(k_{i,t-1}) + b_{i,t} + \frac{m_{i,t-1}}{1+\pi_t} + \frac{t_t}{p_t}$$

$$\eta q_t(k_{i,t} - k_{i,t-1}) + \zeta c_{i,t} = \frac{m_{i,t-1}}{1+\pi_t} + \frac{t_t}{p_t}$$

we can have

$$(1-\eta)q_t(k_{i,t} - k_{i,t-1}) + (1-\zeta)c_{i,t} + m_{i,t} + \frac{R_t b_{i,t-1}}{1+\pi_t} = f(k_{i,t-1}) + b_{i,t}$$

Thus

$$
\begin{aligned}
L = {} & E_0 \sum_{t=0}^{\infty} \beta_i^t u_i(c_{it}) + \sum_{t=0}^{\infty} \beta_i^t \Omega_{i,t} \big[\frac{m_{i,t-1}}{1+\pi_t} + \frac{t_t}{p_t} - \eta q_t(k_{i,t} - k_{i,t-1}) - \zeta c_{i,t} \big] + \\
& \sum_{t=0}^{\infty} \beta_i^t \lambda_{i,t} \big[f(k_{i,t-1}) + b_{i,t} - m_{i,t} - \frac{R_t b_{i,t-1}}{1+\pi_t} - (1-\eta) q_t(k_{i,t} - k_{i,t-1}) - (1-\zeta) c_{i,t} \big]
\end{aligned}
$$

FOCs:

$$
c_{it} : u_i^{'}(c_{it}) - \zeta \Omega_{i,t} - (1-\zeta)\lambda_{i,t} = 0 \tag{8}
$$

$$
m_{i,t} : \beta_i \Omega_{i,t+1} \frac{1}{1+\pi_{t+1}} - \lambda_{i,t} = 0 \tag{9}
$$

$$
b_{i,t} : \lambda_{i,t} - \beta_i \lambda_{i,t+1} R_{t+1} \frac{1}{1+\pi_{t+1}} = 0 \tag{10}
$$

$$
k_{i,t} : -\Omega_{i,t}\eta q_t - \lambda_{i,t}(1-\eta)q_t + \beta_i \Omega_{i,t+1}\eta q_{t+1} + \beta_i \lambda_{i,t+1} f^{'}(k_{i,t}) + \beta_i \lambda_{i,t+1}(1-\eta)q_{t+1} = 0 \tag{11}
$$

For the lenders (type 1 agents). Using (8), (9), (10) to get

$$
\frac{u_1^{'}(c_{1t})}{\zeta R_t + 1 - \zeta} = \beta_1 \frac{u_1^{'}(c_{1t+1})(1+r_{t+1})}{\zeta R_{t+1} + 1 - \zeta} \tag{12}
$$

(8), (9), (10), (11)

$$
u_1^{'}(c_{1t}) q_t \frac{\eta R_t + 1 - \eta}{\zeta R_t + 1 - \zeta} = \beta_1 u_1^{'}(c_{1t+1}) \frac{R_{t+1}\eta q_{t+1} + e^{A_{t+1}} f^{'}(k_{1,t}) + (1-\eta)q_{t+1}}{\zeta R_{t+1} + 1 - \zeta} \tag{13}
$$

For the borrowers, we have

$$
\frac{u_2^{'}(c_{2t})}{\zeta R_t + 1 - \zeta} = \beta_2 \frac{u_2^{'}(c_{2t+1})(1+r_{t+1})}{\zeta R_{t+1} + 1 - \zeta} \tag{14}
$$

$$u_2'(c_{2t})q_t\frac{\eta R_t + 1 - \eta}{\zeta R_t + 1 - \zeta} = \beta_2 u_2'(c_{1t+1})\frac{R_{t+1}\eta q_{t+1} + e^{A_{t+1}}f'(k_{2,t}) + (1-\eta)q_{t+1}}{\zeta R_{t+1} + 1 - \zeta} \tag{15}$$

Substitute (12) into (13)

$$q_t\frac{\eta R_t + 1 - \eta}{1 + r_{t+1}} = R_{t+1}\eta q_{t+1} + e^{A_{t+1}}f'(k_{1,t}) + (1-\eta)q_{t+1} \tag{16}$$

By using the same method, we can easily have

$$q_t\frac{\eta R_t + 1 - \eta}{1 + r_{t+1}} = R_{t+1}\eta q_{t+1} + e^{A_{t+1}}f'(k_{2,t}) + (1-\eta)q_{t+1} \tag{17}$$

The only difference between (16) and (17) are $f'(k_{1,t})$ and $f'(k_{2,t})$ which means $f'(k_{1,t}) = f'(k_{2,t})$

B.3 Existence of Asymptotic Steady State of Model without Collateral Constraints

By (12),
$$\frac{u_1'(c_{1t})}{u_1'(c_{1t+1})} = \beta_1\frac{(1 + r_{t+1})(\zeta R_t + 1 - \zeta)}{\zeta R_{t+1} + 1 - \zeta}$$

By (14)
$$\frac{u_2'(c_{2t})}{u_2'(c_{2t+1})} = \beta_2\frac{(1 + r_{t+1})(\zeta R_t + 1 - \zeta)}{\zeta R_{t+1} + 1 - \zeta}$$

There is no steady state. However there is an asymptotic steady state given a fixed monetary policy where $R_t = R$, $k_{1,t} = k_1$, $k_{2,t} = k_2$, $\pi_t = \pi$ and $\frac{u_1'(c_{1t})}{u_1'(c_{1t+1})} = \beta_1\frac{R}{1+\pi}$, $\frac{u_2'(c_{2t})}{u_2'(c_{2t+1})} = \beta_2\frac{R}{1+\pi}$ and $\beta_1\frac{R}{1+\pi} > 1$ while $\beta_2\frac{R}{1+\pi} < 1$. $c_{1t} \to f(k_1) + f(k_2)$ and $c_{2t} \to 0$.

B.4 Proof of Proposition 4

The Ramsey plan is

$$\max_{c_{1,t},c_{2,t},k_{1,t-1},k_{2,t-1},b_{1t},b_{2t},t_t,m_{1,t},m_{2,t},q_t,p_t,R_t,\pi_t,\Omega_{2,t},\mu_t} E_0\alpha\sum_{t=0}^{\infty}\beta_1^t u_1(c_{1t}) + (1-\alpha)\sum_{t=0}^{\infty}\beta_2^t u_2(c_{i2t})$$

s.t

$$nm_{1,t} + m_{2,t} = n\frac{m_{1,t-1}}{1+\pi_t} + \frac{m_{2,t-1}}{1+\pi_t} + (1+n)\frac{t_t}{p_t}$$

$$\eta q_t(k_{i,t} - k_{i,t-1}) + \zeta c_{i,t} = \frac{m_{i,t-1}}{1+\pi_t} + \frac{t_t}{p_t}$$

$$\frac{u_1'(c_{1t})}{\zeta R_t + 1 - \zeta} = \beta_1 \frac{u_1'(c_{1t+1})(1+r_{t+1})}{\zeta R_{t+1} + 1 - \zeta}$$

$$u_1'(c_{1t})q_t\frac{\eta R_t + 1 - \eta}{\zeta R_t + 1 - \zeta} = \beta_1 u_1'(c_{1t+1})\frac{R_{t+1}\eta q_{t+1} + e^{A_{t+1}}f'(k_{1,t}) + (1-\eta)q_{t+1}}{\zeta R_{t+1} + 1 - \zeta}$$

$$\begin{aligned}\Omega_{2,t}\eta q_t &= \beta_2\Omega_{2,t+1}\left(\frac{\gamma_t q_{t+1}}{R_{t+1}} + \eta q_{t+1} - \frac{1}{1+\pi_{t+1}}(1-\eta)q_t\right)\\ &+ \beta_2^2\Omega_{2,t+2}\frac{e^{A_{t+1}}f_2'(k_{2,t}) + (1-\eta-\gamma_t)q_{t+1}}{1+\pi_{t+2}}\end{aligned}$$

$$u_2'(c_{2t}) = \zeta\Omega_{2,t} + (1-\zeta)\beta_2\Omega_{2,t+1}\frac{1}{1+\pi_{t+1}}$$

$$(1-\eta)q_t(k_{i,t} - k_{i,t-1}) + (1-\zeta)c_{i,t} + m_{i,t} + \frac{R_t b_{i,t-1}}{1+\pi_t} = f(k_{i,t-1}) + b_{i,t}$$

$$R_{t+1}b_{2,t} \le \gamma q_{t+1}(1+\pi_{t+1})k_{2,t}$$

$$b_{2,t} = -nb_{1,t}$$

$$\bar{K} = nk_{1,t} + k_{2,t}$$

$$nm_{1,t} + m_{2,t} = (1+\mu_t)(\frac{nm_{1,t-1}}{(1+\pi_t)} + \frac{m_{2,t-1}}{(1+\pi_t)})$$

$$
\begin{aligned}
L \ = \ & \alpha n E_0 \sum_{t=0}^{\infty} \beta_1^t u_1(c_{1t}) + (1-\alpha) E_0 \sum_{t=0}^{\infty} \beta_2^t u_2(c_{2t}) \\
& + \sum_{t=0}^{\infty} \beta_1^t \theta_t [n \frac{m_{1,t-1}}{1+\pi_t} + \frac{m_{2,t-1}}{1+\pi_t} + (1+n)\frac{t_t}{p_t} - nm_{1,t} - m_{2,t}] \\
& + \sum_{t=0}^{\infty} \beta_1^t \vartheta_{i,t} [\frac{m_{i,t-1}}{1+\pi_t} + \frac{t_t}{p_t} - \eta q_t(k_{i,t} - k_{i,t-1}) - \zeta c_{i,t}] \\
& + \sum_{t=0}^{\infty} \beta_1^t \iota_t [\beta_1 \frac{u_1'(c_{1t+1})(1+r_{t+1})}{\zeta R_{t+1} + 1 - \zeta} - \frac{u_1'(c_{1t})}{\zeta R_t + 1 - \zeta}] \\
& + \sum_{t=0}^{\infty} \beta_1^t \kappa_t [\beta_1 u_1'(c_{1t+1}) \frac{R_{t+1}\eta q_{t+1} + e^{A_{t+1}} f'(k_{1,t}) + (1-\eta)q_{t+1}}{\zeta R_{t+1} + 1 - \zeta} \\
& - u_1'(c_{1t}) q_t \frac{\eta R_t + 1 - \eta}{\zeta R_t + 1 - \zeta}] \\
& + \sum_{t=0}^{\infty} \beta_1^t \lambda_t [\beta_2 \Omega_{2,t+1} \left(\frac{\gamma_t q_{t+1}}{R_{t+1}} + \eta q_{t+1} - \frac{1}{1+\pi_{t+1}}(1-\eta)q_t \right) \\
& + \beta_2^2 \Omega_{2,t+2} \frac{e^{A_{t+1}} f_2'(k_{2,t}) + (1-\eta-\gamma_t)q_{t+1}}{1+\pi_{t+2}} - \Omega_{2,t}\eta q_t] \\
& + \sum_{t=0}^{\infty} \beta_1^t \varpi_t [u_2'(c_{2t}) - \zeta \Omega_{2,t} - (1-\zeta)\beta_2 \Omega_{2,t+1} \frac{1}{1+\pi_{t+1}}] \\
& + \sum_{t=0}^{\infty} \beta_1^t \varkappa_{i,t} [f(k_{i,t-1}) + b_{i,t} - (1-\eta)q_t(k_{i,t} - k_{i,t-1}) - (1-\zeta)c_{i,t} - m_{i,t} - \frac{R_t b_{i,t-1}}{1+\pi_t}] \\
& + \sum_{t=0}^{\infty} \beta_1^t \nu_t [\gamma_t q_{t+1}(1+\pi_{t+1})k_{2,t} - R_{t+1}b_{2,t}] \\
& + \sum_{t=0}^{\infty} \beta_1^t \xi_t [\frac{b_{2,t}}{p_t} + n\frac{b_{1,t}}{p_t}] \\
& + \sum_{t=0}^{\infty} \beta_1^t \varsigma_t [\bar{K} - nk_{1,t} - k_{2,t}] \\
& + \sum_{t=0}^{\infty} \beta_1^t \varphi_t [(1+\mu_t)(\frac{nm_{1,t-1}}{1+\pi_t} + \frac{m_{2,t-1}}{1+\pi_t}) - (nm_{1,t} + m_{2,t})]
\end{aligned}
$$

$$
b_{1t} : \varkappa_{1,t} - \beta_1 \varkappa_{1,t} \frac{R_{t+1}}{(1+\pi_{t+1})} + \xi_t n = 0
$$

$$
b_{2t} : \varkappa_{2,t} - \beta_1 \varkappa_{2,t+1} \frac{R_{t+1}}{(1+\pi_{t+1})} - \nu_{t+1}R_{t+1} + \xi_t = 0
$$

$$
q_t : -\vartheta_{1,t}\eta(k_{1,t} - k_{1,t-1}) - \vartheta_{2,t}\eta(k_{2,t} - k_{2,t-1})
$$

$$
\begin{aligned}
& -\kappa_t [u_1'(c_{1t}) \frac{\eta R_t + 1 - \eta}{\zeta R_t + 1 - \zeta}] + \kappa_{t-1} [u_1'(c_{1t}) \frac{R_t \eta + (1-\eta)}{\zeta R_t + 1 - \zeta}] \\
& + \lambda_t [-\beta_2 \Omega_{2,t+1} \frac{1}{1+\pi_{t+1}}(1-\eta) - \Omega_{2,t}\eta] \\
& + \frac{1}{\beta_1} \lambda_{t-1} [\beta_2 \Omega_{2,t} \left(\frac{\gamma_{t-1}}{R_t} + \eta \right) + \beta_2^2 \Omega_{2,t+1} \frac{(1-\eta-\gamma_{t-1})}{1+\pi_{t+1}} + \nu_{t-1}\gamma_{t-1}(1+\pi_t)k_{2,t-1}]
\end{aligned}
$$

Find F.O.Cs and study the steady state,

$$b_1 : \xi n = 0$$

$$b_2 : -\nu\frac{1+\mu}{\beta} + \xi = 0$$

which means

$$\nu = 0$$

So

$$q : \lambda[-\Omega_2\eta - \beta_2\Omega_2\frac{1}{1+\pi}(1-\eta)] + \frac{1}{\beta_1}\lambda[\beta_2\Omega_2\left(\frac{\gamma}{R} + \eta\right) + \beta_2^2\Omega_2\frac{(1-\eta-\gamma)}{1+\pi}] = 0$$

turns into

$$-\beta_1\eta - \beta_1\beta_2(1-\eta) + \beta_2\left(\beta_1\gamma + \eta(1+\pi)\right) + \beta_2^2(1-\eta-\gamma) = 0$$

which means

$$1 + \pi = \frac{\beta_2\left(\gamma + \eta - 1\right)}{\eta}$$

B.5 Model without Redistribution of Cash

If we want to check whether it is the redistribution of money which lead to the most effect in our paper, we can change several settings of our model and get a model without any redistribution effect of cash. The first change is cash in advance constraints. We have new cash in advance constraints here

$$\eta q_t(k_{i,t} - k_{i,t-1}) + \zeta c_{i,t} = \frac{m_{i,t-1}}{1+\pi_{t-1}} + \frac{t_{i,t}}{p_t}$$

The difference between this one and the old one is we allow different types of households get different money transfer from government.

When in the government part, monetary policy now change from the original one to

$$M_t = M_{t-1}(1 + mu_t) = M_{t-1} + nt_{1,t} + t_{2,t}$$

$$t_{1,t} = (1 + \mu_t) * m_{1,t-1}$$

Following the same method, we can easily find that the optimal monetary policy at steady state is still $1 + \pi = \frac{\beta_2(\gamma+\eta-1)}{\eta}$.

We can compare the monetary policy in short term with the model in our paper. The red line is for original model and the green line is for model without redistribution of money. We can see it is close with each others which means the redistribution of money is not important and the redistribution through inflation and cost dominates.
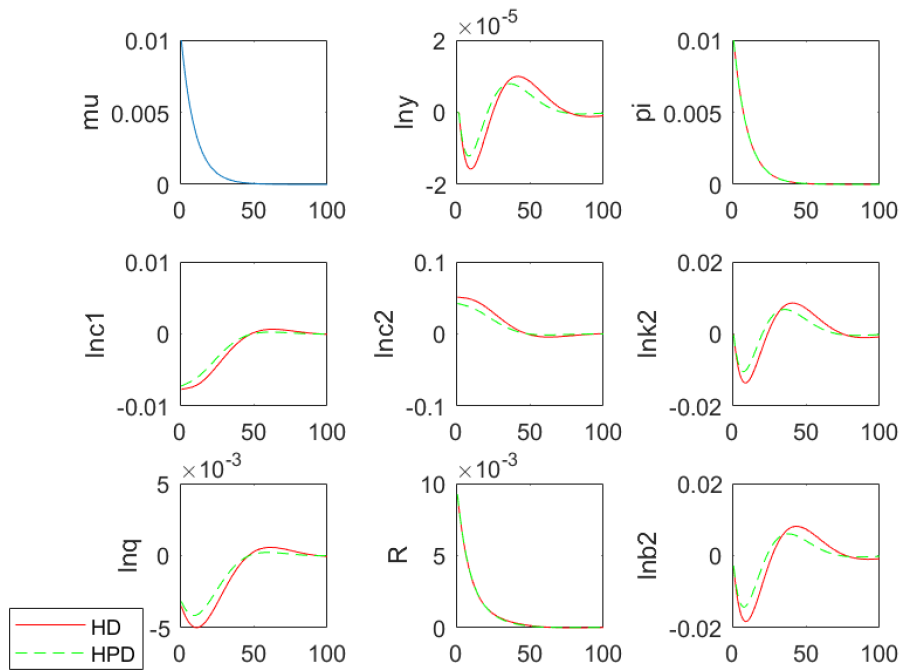

Figure 2: Comparison of two models